REVSTAT – Statistical Journal
Volume 12, Number 3, November 2014, 221–245

ON THE IMPACT OF FALSELY ASSUMING I.I.D. OUTPUT IN THE PROBABILITY OF MISLEADING SIGNALS

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Received: February 2013 Accepted: September 2013

Abstract:

- Misleading signals (MS) are valid alarms which correspond to the misinterpretation of a shift in the process mean (resp. variance) as a shift in the process variance (resp. mean), when we deal with simultaneous schemes for these two parameters. MS can be fairly frequent, as reported by some authors, and occur for instance when:
  - the individual chart for the mean triggers a signal before the one for the variance, even though the process mean is on-target and the variance is off-target; or
  - the individual chart for the variance triggers a signal before the one for the mean, although the variance is in-control and the process mean is out-of-control.

  This paper illustrates how (un)reliable are the traditional simultaneous Shewhart- and EWMA-type schemes in identifying which parameter has changed, under the false assumption of independence, namely when the output process within each sample follows AR(1), AR(2) or ARMA (1,1) models. This is done by means of Monte Carlo simulation and the estimation of the probability of a misleading signal (PMS).
  Finally, we go on to compare these estimates of PMS with the values of the PMS of simultaneous Shewhart- and EWMA-type residual schemes whose control statistics take into account the autocorrelation structure of the output process.

Key-Words:

- *statistical process control; misleading signals; time series; simultaneous residual schemes.*

AMS Subject Classification:

- 62P30, 60G99.

1. THE PHENOMENON OF MISLEADING SIGNALS

In most monitoring applications, we assume that the quality characteristic is an absolutely continuous random variable with a normal distribution with mean μ and variance σ^2. Quality control charts are graphical SPC tools whose main purpose is to detect (removable) special or assignable causes responsible for changes in μ and σ^2. Standard practice is to run two individual charts at the same time, one for μ and another one for σ^2. The resulting scheme is known as a simultaneous scheme and it provides a way to satisfy Shewhart's dictum that proper process control implies monitoring both location and dispersion.

When we use a simultaneous scheme, the quality characteristic is deemed to be out-of-control whenever a signal is triggered by either individual chart: a signal suggests a potential change in μ, in σ^2 or in both μ and σ^2. Moreover, it is expected that the chart for the mean will help us detect increases or decreases in μ from a target value μ_0 and that the chart for the variance will assist us in the detection of increases in σ^2 from an in-control value σ_0^2. However, it has been pointed out by some authors (e.g. [21], [10] and [18]) that the misidentification of the parameter that has changed can occur frequently, which means that a shift in μ can be misinterpreted as a shift in σ^2 and vice-versa. [21] termed these two events as misleading signals (MS) and [10] systematized them and only considered MS of types III and IV:

- the individual chart for μ triggers a signal before the one for σ^2, although the process mean is on-target and the variance is off-target; and
- the individual chart for σ^2 triggers a signal before the one for μ, even though the process variance is on-target and the mean is off-target.

Now, note that special or assignable causes on the chart for μ can differ from those on the chart for σ^2: for instance, cyclic patterns in $\bar{X}$-charts may result from systematic changes in temperature or regular rotation of operators/machines, whereas S^2-charts reveal cycles because of maintenance schedules or tool wear ([9, pp. 189–190]). Furthermore, the diagnostic and correction procedures that follow a signal can differ depending on which chart triggers the alarm, as mentioned by [11] and [8]. Therefore, the occurrence of a MS can lead to an inappropriate diagnose and to unnecessary correction measures and hence to an increase in production and inspection costs.

2. EXISTING WORK

The main question regarding misleading signals should not be whether they happen or not, but rather how frequently they occur, as pointed out by [11].

Unsurprisingly, the probability of a misleading signal (PMS) should be considered as an additional performance measure of simultaneous schemes for μ and σ^2.

The behavior of the PMS of types III and IV has been addressed for i.i.d. and Gaussian output by a few authors ([10], [18], [12], [19] and [11]). For example, the numerical results in [12] and [11] suggest that simultaneous Shewhart-type schemes compare unfavorably to their EWMA counterparts and that the values of both PMS are far from negligible, specially for small and moderate shifts in μ and σ^2.

The study of the phenomenon of MS has been extended by [2], [8], [14] and [15] to the following change point model, proposed by [7] and [6] and dealing with autocorrelated output. Let us denote by $\{Y_{i,j}\}$ the target process, where i represents the sample number and j the number of the observation within the sample. Samples have fixed size n, are independent and represented by $(Y_{i,1}, ..., Y_{i,n})$. However, we shall assume that $\{Y_{i,1}, ..., Y_{i,n}\}$ follows a (weakly) stationary Gaussian process with known mean μ_0 and known autocovariance function $\{\gamma_0, \gamma_1, ..., \gamma_{n-1}\}$, for every i. The observed process, $\{X_{i,j}\}$, is related to the target process as follows:

$$X_{i,j} \;=\; \mu_0 + \delta\,\gamma_0 + \theta\,(Y_{i,j} - \mu_0)\;, \qquad i = 1, 2, \ldots\;, \tag{2.1}$$

where $\delta = [E(X_{i,j}) - \mu_0]/\sqrt{\gamma_0}$ (resp. $\theta = \sqrt{V(X_{i,j})/\gamma_0}$) represents the magnitude of the shift in the process mean (resp. standard deviation). As put by [8], the assumption of independent samples but autocorrelated output within each sample is rather reasonable in SPC because the intervals between successive samples are significantly large when compared to the time required to take a sample, resulting in negligible correlation between samples and considerable correlation within each sample.

There are essentially three approaches to monitor shifts in the mean and variance of the observed process and they play a major role in the performance of the simultaneous schemes and, obviously, on the PMS. We could plot the sample mean and variance of each of the original data in a traditional simultaneous scheme, however, with readjusted control limits to account for the autocorrelation; the resulting scheme is called a modified simultaneous scheme ([7] and [6]). Alternatively, we could plot the sample mean and variance of the residuals instead of the original data, in a traditional simultaneous scheme, i.e., use what is called a simultaneous residual scheme ([7], [6] and [8]). Lastly, we could ignore the autocorrelation structure and assume the output is i.i.d. within each sample and use the traditional simultaneous schemes.

Results by [8], [2], [14] and [15] suggest that the presence of autocorrelation can have a significant impact in the PMS of simultaneous Shewhart and EWMA residual schemes for the process mean and variance of stationary processes.

[8] and [2] showed that the PMS of Type III is not affected by the autoregressive parameter and larger nonnegative values of this parameter are associated to more frequent MS of Type IV, when dealing with simultaneous Shewhart- and EWMA-type residual schemes for the mean and the variance of AR(1) output. [14] used stochastic ordering to prove that the PMS of Type IV of simultaneous Shewhart (resp. EWMA) residual schemes for the process mean and variance of stationary AR(1) output increases with the autoregressive parameter in the interval $(-1, 1)$ (resp. (0,1)). [15] is an obvious extension of [14] to general stationary Gaussian processes, such as AR(2) and ARMA(1,1) models, and it also identified regions where the PMS of Type IV is a monotonous function of the parameters of these models. In addition to this, [8] showed how unreliable are the traditional simultaneous Shewhart and EWMA schemes in identifying which parameter has changed under the false assumption of i.i.d. output, when we are in fact dealing with stationary AR(1) output.

In the present paper, we recall some of these results for AR(1) output and we extend these investigations to AR(2) and ARMA(1,1) processes with a few unexpected results. All the estimates of PMS were obtained via an extensive Monte Carlo simulation study and we go on to compare them to the PMS values associated with simultaneous residual schemes. But before proceeding to this study of the impact of falsely assuming i.i.d. output on the PMS, we shall briefly describe simultaneous residual schemes for autocorrelated output in the next section.

3. SIMULTANEOUS RESIDUAL SCHEMES AND PMS

Residual charts ([1]) can prevent the process mean and variance of autocorrelated output from wandering too far from their targets. Besides that, these charts are theoretically very appealing because their control statistics take the autocorrelation explicitly into account, and reduce the monitoring problem to the well-known case of detecting shift in the mean and variance of i.i.d. output ([23, p. 63]). Moreover, since control charts are ultimately used by non-statisticians, we favor "one fits all" procedures, such as residual charts, that are easily understood and can be applied to most industrial processes.

The control statistics of the individual residuals charts for the process mean and variance of a stationary Gaussian process may be defined in terms of standardized residuals ([8]), such as the following ones

$$\begin{aligned} \hat{\varepsilon}_{i,j} &= \frac{X_{i,j} - \hat{X}_{i,j}}{\sqrt{V_{\delta=0,\theta=1}(X_{i,j} - \hat{X}_{i,j})}} \\ &= \theta\, \hat{\epsilon}_{i,j} + \delta \sqrt{\gamma_0}\, b_j \ , \end{aligned} \tag{3.1}$$

where: $V_{\delta=0,\theta=1}(X_{i,j} - \hat{X}_{i,j})$ represents the in-control variance of the residuals of the output process; $\hat{\epsilon}_{i,j} \sim_{i.i.d.} \mathcal{N}(0,1)$ are the standardized residuals of the target process; $\mathbf{b} = (b_1, ..., b_n)$ is the vector of the b_j s, which are functions of $\gamma_0, \gamma_1, ..., \gamma_{n-1}$ that can be recursively obtained by using the Durbin–Levinson algorithm ([3, p. 169]). If the (fitted) model is valid, the standardized residuals are independent normal r.v. and the sample mean and variance of these residuals,

$$\bar{\hat{\varepsilon}}_i = \frac{1}{n} \sum_{j=1}^{n} \hat{\varepsilon}_{i,j} , \tag{3.2}$$

$$\hat{S}_i^2 = \frac{1}{n-1} \sum_{j=1}^{n} (\hat{\varepsilon}_{i,j} - \bar{\hat{\varepsilon}}_i)^2 , \tag{3.3}$$

are independent r.v. such that

$$\bar{\hat{\varepsilon}}_i \sim_{i.i.d.} \mathcal{N}\left(\frac{\delta\sqrt{\gamma_0}}{n} \sum_{j=1}^{n} b_j , \frac{\theta^2}{n}\right) , \tag{3.4}$$

$$\frac{(n-1)\,\hat{S}_i^2}{\theta^2} \sim_{i.i.d.} \chi^2_{n-1,\nu} , \tag{3.5}$$

where $\chi^2_{n-1,\nu}$ denotes the noncentral χ^2-distribution with $n-1$ degrees of freedom and noncentrality parameter equal to

$$\nu = \left(\frac{\delta}{\theta}\right)^2 \gamma_0 \left(\sum_{j=1}^{n} b_j^2 - n\bar{b}^2\right) . \tag{3.6}$$

The control limits of the individual charts for μ and σ^2 that constitute the simultaneous residual scheme do not depend on the underlying in-control observed process — be it i.i.d. or autocorrelated —, coincide with the ones of traditional individual charts for the mean and variance of i.i.d. processes, and are listed in Table 1 for convenience and were previously adopted by [8].

By capitalizing on the distributional properties of $\bar{\hat{\epsilon}}_i$ and $\hat{S}_i^2$ we can conclude that the run lengths of the individual Shewhart-type residual charts for μ and σ^2, $RL_{S-\mu}(\delta, \theta, \mathbf{b})$ and $RL_{S-\sigma}(\delta, \theta, \mathbf{b})$, and the run length of the simultaneous Shewhart residual scheme, $RL_{S-\mu,\sigma}(\delta, \theta, \mathbf{b})$, have geometric distributions with parameters say $\xi_{S-\mu}(\delta, \theta, \mathbf{b})$, $\xi_{S-\sigma}(\delta, \theta, \mathbf{b})$ and $\xi_{S-\mu,\sigma}(\delta, \theta, \mathbf{b})$, where $\xi_{S-\mu,\sigma}(\delta, \theta, \mathbf{b}) = \xi_{S-\mu}(\delta, \theta, \mathbf{b}) + \xi_{S-\sigma}(\delta, \theta, \mathbf{b}) - \xi_{S-\mu}(\delta, \theta, \mathbf{b}) \times \xi_{S-\sigma}(\delta, \theta, \mathbf{b})$ because a simultaneous residual scheme triggers a signal as soon as a signal is observed on either constituent charts. The Markov chain approach ([4]) provides approximations to the distributions of the run lengths $RL_{E-\mu}(\delta, \theta, \mathbf{b})$, $RL_{E-\sigma}(\delta, \theta, \mathbf{b})$ and $RL_{E-\mu,\sigma}(\delta, \theta, \mathbf{b})$. As a consequence we can provide exact expressions (resp. approximate values) for the average run length (ARL) or any other RL related performance measure, such as the PMS of simultaneous Shewhart (resp. EWMA)

Table 1: Control statistics and limits of the individual Shewhart- $(S-\mu, S-\sigma)$ and EWMA-type $(E-\mu, E-\sigma)$ residual charts for μ and σ^2.

Control statistics	**Control limits**
$\bar{\hat{\varepsilon}}_i$	$LCL_{S-\mu} = -\frac{\gamma_{S-\mu}}{\sqrt{n}}$ $UCL_{S-\mu} = -LCL_{S-\mu}$
$\hat{S}_i^2$	$LCL_{S-\sigma} = 0$ $UCL_{S-\sigma} = 1 + \gamma_{S-\sigma}\sqrt{\frac{2}{n-1}}$
$Z_{\bar{\hat{\varepsilon}},i} = \begin{cases} E(\bar{\hat{\varepsilon}}) = 0\,, & i = 0\,, \\ (1-\lambda_\mu)\, Z_{\bar{\hat{\varepsilon}},i-1} + \lambda_\mu\, \bar{\hat{\varepsilon}}_i\,, & i = 1, ... \end{cases}$	$LCL_{E-\mu} = -\gamma_{E-\mu}\sqrt{\frac{\lambda_\mu}{n(2-\lambda_\mu)}}$ $UCL_{E-\mu} = -LCL_{E-\mu}$
$Z_{\hat{S}^2,i} = \begin{cases} E(\hat{S}^2) = 1\,, & i = 0\,, \\ (1-\lambda_\sigma)\, Z_{\hat{S}^2,i-1} + \lambda_\sigma\, \hat{S}_i^2\,, & i = 1, ... \end{cases}$	$LCL_{E-\sigma} = 0$ $UCL_{E-\sigma} = 1 + \gamma_{E-\sigma}\sqrt{\frac{2\,\lambda_\sigma}{(n-1)\,(2-\lambda_\sigma)}}$

residual schemes. In fact, if we focus on the detection of downward and upward shifts in μ and upward shifts in σ^2, then the two PMS can be simply written as

$$\begin{aligned} \mathrm{PMS_{III}}(\theta, \mathbf{b}) &= P\big[RL_\mu(0,\theta,\mathbf{b}) < RL_\sigma(0,\theta,\mathbf{b})\big] \\ &= \sum_{i=1}^{+\infty} P\big[RL_\mu(0,\theta,\mathbf{b}) = i\big] \times P\big[RL_\sigma(0,\theta,\mathbf{b}) > i\big]\,, \qquad \theta > 1\ , \end{aligned} \tag{3.7}$$

$$\begin{aligned} \mathrm{PMS_{IV}}(\delta, \mathbf{b}) &= P\big[RL_\sigma(\delta,1,\mathbf{b}) < RL_\mu(\delta,1,\mathbf{b})\big] \\ &= \sum_{i=1}^{+\infty} P\big[RL_\sigma(\delta,1,\mathbf{b}) = i\big] \times P\big[RL_\mu(\delta,1,\mathbf{b}) > i\big]\,, \qquad \delta \neq 0\ , \end{aligned} \tag{3.8}$$

where RL_μ and RL_σ denote the RL of the individual Shewhart or EWMA-type charts for μ and σ^2, respectively. We ought to mention that a relative error of 10^{-6} is considered in the truncation of the series defining $\mathrm{PMS_{III}}(\theta, \mathbf{b})$ and $\mathrm{PMS_{IV}}(\delta, \mathbf{b})$, whenever we need to calculate approximate values of these two performance measures. For more details on the exact and approximate distributions of these RL and on the exact and approximate values of the PMS, please refer to [8].

4. THE IMPACT OF FALSELY ASSUMING INDEPENDENCE ON THE PMS

In this section, we shall ignore the autocorrelation structure, assume that the output is i.i.d. within each sample and use traditional individual charts to detect shifts in μ and upward shifts in σ^2. The control limits of these charts coincide with the ones of the individual residual charts (see Table 2). However, the control statistics depend on the sample mean and variance of the standardized output,

$$\bar{X}_i^* = \frac{1}{n} \sum_{j=1}^{n} \frac{X_{i,j} - \mu_0}{\sqrt{\gamma_0}} , \tag{4.1}$$

$$(S_i^*)^2 = \frac{1}{n-1} \sum_{j=1}^{n} \frac{(X_{i,j} - \bar{X}_i)^2}{\gamma_0} , \tag{4.2}$$

not on the sample mean and variance of the standardized residuals. Suffice to say that $\bar{X}_i^*$ and $(S_i^*)^2$ are the control statistics of the traditional Shewhart-type charts for μ and σ^2 ($S^* - \mu$ and $S^* - \sigma$). As for the traditional EWMA-type charts ($E^* - \mu$ and $E^* - \sigma$), they make use of the statistics

$$Z_{\bar{X}^*,i} = \begin{cases} 0 , & i = 0 , \\ (1-\lambda_\mu)\, Z_{\bar{X}^*,i-1} + \lambda_\mu \bar{X}_i^* , & i = 1, \dots , \end{cases} \tag{4.3}$$

$$Z_{(S^*)^2,i} = \begin{cases} 1 , & i = 0 , \\ (1-\lambda_\sigma)\, Z_{(S^*)^2,i-1} + \lambda_\sigma (S_i^*)^2 , & i = 1, \dots . \end{cases} \tag{4.4}$$

Should the output be i.i.d. or simultaneous residual schemes for the mean and variance of autocorrelated output are at use, we would be able to provide exact expressions (resp. approximations) for the PMS in the Shewhart (resp. EWMA) case, as seen in the previous section. Be that as it may, in the presence of autocorrelation, the statistics $\bar{X}_i^*$ and $(S_i^*)^2$ are no longer independent r.v., and therefore we have to rely on Monte Carlo simulation to obtain estimates of the PMS, when the output process within each sample, follows an AR(1), AR(2) or an ARMA(1,1) model.

For illustration purposes, we considered the target process, $(Y_{i,1}, ..., Y_{i,n})$ for each i ($i = 1, ..., rep$), drawn from a Gaussian stationary process with zero mean ($\mu_0 = 0$) and unit variance ($\gamma_0 = 1$), where the number of replications is equal to $rep = 10^6$ for each set of parameter values. Furthermore, we simulated samples of size $n = 5$ of this in-control process, obtained the out-of-control process and the observed values of the control statistics, compared the latter with the control limits and counted the number of misleading signals and the number of signals triggered by the simultaneous schemes and estimated the corresponding PMS. In addition to that, we have taken: $\lambda_\mu = \lambda_\sigma = \lambda = 1, 0.05$ (allowing the comparison between Shewhart- and EWMA-type schemes); $\theta = 1.02, 1.10, 1.20$ (PMS of Type III); $\delta = 0.05, 0.50, 1.00$ (PMS of Type IV). Moreover, the critical values $\gamma_{S-\mu}$, $\gamma_{S-\sigma}$, $\gamma_{E-\mu}$ and $\gamma_{E-\sigma}$ were calculated in such way that the in-control average run length (ARL) of both the individual traditional charts for μ and σ are approximately the same, i.e. $ARL_\mu(0, 1, \mathbf{b}) = ARL_\sigma(0, 1, \mathbf{b})$, and the ARL of the simultaneous scheme is approximately equal to 500 samples, that is $ARL_{\mu,\sigma}(0, 1, \mathbf{b}) = 500$; the resulting critical values and the corresponding in-control ARL are summarized in Table 2 and coincide with the ones in [8]. Please bear in mind that, when dealing with Markov approximations, we considered 101 transient states to determine these critical values and all the RL related measures.

Table 2: Critical values for the individual Shewhart ($\lambda = 1$) and EWMA charts.

γ_μ	γ_σ	$ARL_\mu(0,1,\mathbf{b})$	$ARL_\mu(0,1,\mathbf{b})$	$ARL_{\mu,\sigma}(0,1,\mathbf{b})$	λ
3.2904	5.1144	999.550	999.495	500.011	1
2.8817	2.9103	986.202	986.162	499.641	0.05

4.1. AR(1) model

The AR(1) model is usually reported as the most frequently encountered in practice ([23, p. 10]). The process $\{Y_{i,j}\}$ follows a stationary Gaussian AR(1) model with mean μ_0, variance $\gamma_0 = \sigma_0^2$ and autoregressive parameter ϕ, for each i, if

$$Y_{i,j} = \mu_0 + \phi(Y_{i,j-1} - \mu_0) + \varepsilon_{i,j} \ , \tag{4.5}$$

where: ϕ is a constant satisfying $-1 < \phi < 1$; and $\{\varepsilon_{i,j}\}$ is a sequence of disturbances such that $\varepsilon_{i,j} \sim_{i.i.d.} \mathcal{N}(0, \sigma_\varepsilon^2)$, with $\sigma_\varepsilon^2 = (1 - \phi^2) \times \sigma_0^2$.

If we use simultaneous Shewhart- and EWMA-type residual schemes then we can provide exact and approximate values of PMS of Type III (resp. IV); these results can be found in Table 3 (resp. in the center of Table 4). As previously noted by [8] and illustrated by Table 3, the PMS of Type III does not depend on ϕ. In fact, a close inspection of the noncentrality parameter ν, the probabilities $\xi_{S-\mu}(\delta, \theta, \mathbf{b})$, $\xi_{S-\sigma}(\delta, \theta, \mathbf{b})$, etc. leads to the conclusion that these parameters do not depend on $\mathbf{b}$ — when $\delta = 0$ —, thus $\mathrm{PMS_{III}}(\theta, \mathbf{b}) := \mathrm{PMS_{III}}(\theta)$ for any Gaussian stationary model. Table 3 (resp. 4) also shows that $\mathrm{PMS_{III}}(\theta)$ (resp. $PMS_{\mathrm{IV}}(\delta, \phi)$) can be larger than 0.47 (resp. 0.49), for very small shifts in σ^2 (resp. μ), while at the same time reinforcing that the simultaneous Shewhart residual scheme seems to have larger PMS of Type III (resp. Type IV) than its EWMA analog. It should also be noted that $\mathrm{PMS_{IV}}(\delta, \phi)$ appears to increase with $\phi \in (0, 1)$, as already referred by [8].

Now, we investigate what happens to both PMS if the autocorrelation structure is not recognized or ignored and traditional control charts are used when $\phi \in (-1, 1)$. A reasonably large set of estimates of the PMS of types III and IV when autocorrelation is disregarded can be found in Table 4, along with values of $\mathrm{PMS_{IV}}(\delta, \phi)$ when adequate simultaneous Shewhart and EWMA residual schemes were used instead of the traditional ones. Even though the values in Table 4 refer to $\pm\phi = 0, 0.3, 0.5, 0.7, 0.9, 0.95$, figures 1 through 4 were drawn considering $\pm\phi = 0(0.05)0.95(0.01)0.99$; these estimates will be made available to those who are interested and request them from the authors. As in [8], we obtained estimates of PMS of types III and IV that are close to the corresponding values of PMS when simultaneous residual schemes are at use, for $\phi = 0$, as illustrated by Table 4 and by the grey and black lines intersecting at $\phi = 0$ in figures 1–4.

Table 3: PMS of Type III of simultaneous Shewhart ($\lambda = 1$) and EWMA residual schemes.

θ	$\text{PMS}_{\text{III}}(\theta)$	λ
1.02	0.475786	1
	0.343880	0.05
1.10	0.397714	1
	0.100265	0.05
1.20	0.331373	1
	0.042865	0.05

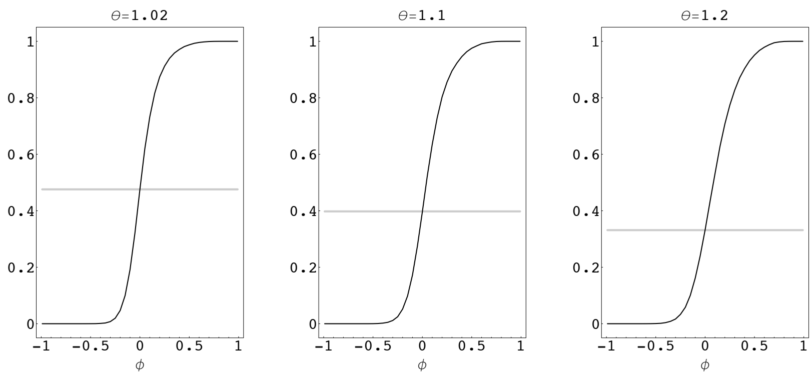

Figure 1: AR(1) model, Shewhart — $\text{PMS}_{\text{III}}(\theta)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type III (traditional simultaneous scheme, black line).

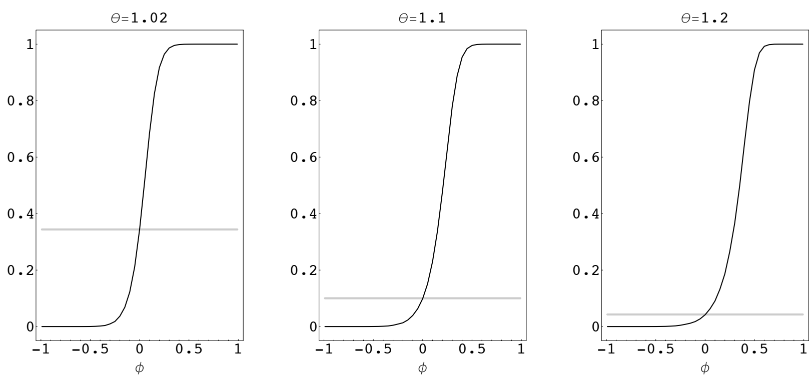

Figure 2: AR(1) model, EWMA — $\text{PMS}_{\text{III}}(\theta)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type III (traditional simultaneous scheme, black line).

Table 4: AR(1) model — estimates of PMS of Type III of traditional simultaneous scheme; $\text{PMS}_{\text{IV}}(\delta, \phi)$ of simultaneous residual scheme; estimates of PMS of Type IV of traditional simultaneous scheme.

		$\phi \in (-1,1)$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
θ	1.02	0.000000	0.000000	0.000000	0.000110	0.007510	0.475890	0.939740	0.987590	0.999240	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000250	0.009400	0.344270	0.986560	0.999830	1.000000	1.000000	1.000000	0.05
	1.10	0.000000	0.000000	0.000000	0.000170	0.011470	0.396250	0.895130	0.975500	0.997660	1.000000	1.000000	1
		0.000000	0.000000	0.000010	0.000230	0.004680	0.098620	0.780360	0.995280	0.999970	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000580	0.016570	0.332010	0.827120	0.951150	0.994980	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000140	0.002480	0.041700	0.366390	0.909000	0.999800	1.000000	1.000000	0.05
δ	0.05	0.245605	0.331756	0.429811	0.457404	0.470828	0.481977	0.488703	0.492034	0.494900	0.497702	0.498534	1
		0.021403	0.040098	0.109687	0.169903	0.221933	0.288653	0.346313	0.381892	0.417014	0.455573	0.467676	0.05
	0.50	0.000411	0.003322	0.009211	0.017601	0.030661	0.061967	0.113516	0.165244	0.240161	0.363509	0.414297	1
		0.004417	0.002008	0.000750	0.000930	0.001422	0.002907	0.006195	0.010799	0.020925	0.055933	0.085086	0.05
	1.00	0.000000	0.000000	0.000620	0.001398	0.002359	0.005825	0.016431	0.035020	0.081256	0.229454	0.323318	1
		0.002409	0.012133	0.000422	0.000158	0.000141	0.000262	0.000794	0.002010	0.006279	0.032854	0.064217	0.05
δ	0.05	1.000000	1.000000	1.000000	0.999940	0.992360	0.483350	0.050420	0.009680	0.000500	0.000000	0.000000	1
		1.000000	1.000000	0.999770	0.992400	0.919140	0.288740	0.003400	0.000050	0.000010	0.000000	0.000000	0.05
	0.50	0.999960	0.999980	0.998590	0.953870	0.577890	0.061520	0.011340	0.003560	0.000350	0.000000	0.000000	1
		0.180890	0.158220	0.090350	0.043940	0.017100	0.003350	0.000370	0.000010	0.000000	0.000000	0.000000	0.05
	1.00	0.788930	0.745620	0.419080	0.137540	0.036130	0.005740	0.002070	0.000840	0.000120	0.000000	0.000000	1
		0.061330	0.049940	0.021770	0.007360	0.002170	0.000310	0.000040	0.000010	0.000000	0.000000	0.000000	0.05

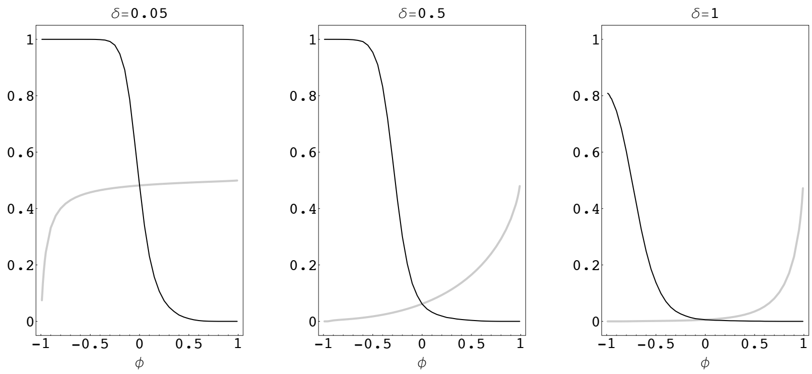

Figure 3: AR(1) model, Shewhart — $\text{PMS}_{\text{IV}}(\delta, \phi)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type IV (traditional simultaneous scheme, black line).

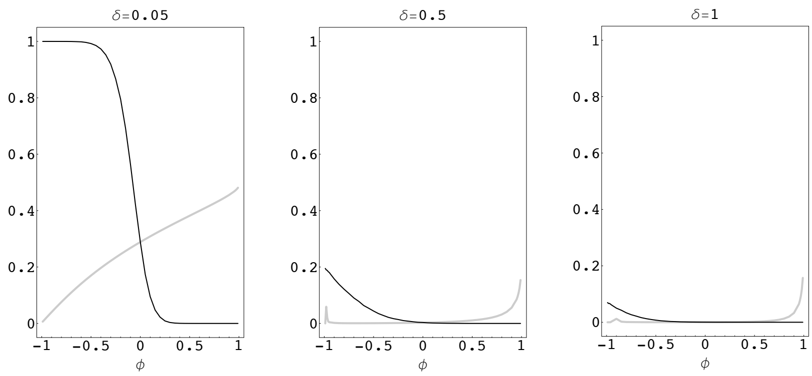

Figure 4: AR(1) model, EWMA — $\text{PMS}_{\text{IV}}(\delta, \phi)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type IV (traditional simultaneous scheme, black line).

When we neglect the autocorrelation structure, the estimates of the PMS of Type III increase from 0 to 1 with ϕ, even though $\text{PMS}_{\text{III}}(\theta)$ does not exceed 0.5 or depend on ϕ when simultaneous residual schemes are at use, as figures 1 and 2 portray quite vividly. Besides that, it is apparent from figures 3 and 4 that the estimates of PMS of Type IV seem to decrease with ϕ, whereas for simultaneous residual schemes $\text{PMS}_{\text{IV}}(\delta,\phi)$ tends to increase with ϕ (see Table 4). Additionally, the PMS of types III and IV are very sensitive to autocorrelation, for instance, we got for the simultaneous EWMA scheme:

- $\text{PMS}_{\text{III}}(1.02) = 0.343880$, still the corresponding estimates are 0.000110 and 0.987590, for $\phi = -0.5$ and $\phi = 0.5$;
- $\text{PMS}_{\text{IV}}(0.05, -0.5) = 0.169903$ and $\text{PMS}_{\text{IV}}(0.05, 0.5) = 0.381892$, while the estimated values are 0.992400 and 0.000050.

It should be also added that the values in tables 3 and 4 and the graphs in figures 1–4 suggest that replacing the traditional Shewhart with traditional EWMA charts only offers improvement with regard to MS of Type IV (for all values of ϕ), even though both $\text{PMS}_{\text{III}}(\theta)$ and $\text{PMS}_{\text{IV}}(\delta,\phi)$ seem to decrease when a simultaneous EWMA residual scheme takes the place of a simultaneous Shewhart residual scheme.

4.2. AR(2) model

The AR(2) process was originally used by G.U. Yule in 1927 to describe the behavior of a simple pendulum and since then it has been widely used to describe a variety of phenomena, namely occurring in engineering and other related fields ([22]) such as industry. Let us recall that the process $\{Y_{i,j}\}$ follows a stationary AR(2) model with mean μ_0, variance $\gamma_0 = \sigma_0^2$ and parameters ϕ_1 and ϕ_2, for each i, if

$$Y_{i,j} = \mu_0 + \phi_1(Y_{i,j-1} - \mu_0) + \phi_2(Y_{j,i-2} - \mu_0) + \varepsilon_{i,j} \;, \tag{4.6}$$

where: the parameters ϕ_1 and ϕ_2 lie in a triangular region restricted by $-1<\phi_2<1$, $\phi_1 + \phi_2 < 1$ and $\phi_2 - \phi_1 < 1$; and the innovations satisfy $\varepsilon_{i,j} \sim_{i.i.d.} \mathcal{N}(0,\sigma_\varepsilon^2)$, with $\sigma_\varepsilon^2 = \frac{(1+\phi_2)\,[(1-\phi_2)^2-\phi_1^2]}{1-\phi_2} \times \sigma_0^2$.

The investigations on the impact of falsely assuming i.i.d. output — instead of an AR(2) model — in the PMS of types III and IV led to some interesting results.

Firstly, note that the graphs in Figure 5 (resp. 6) were restricted to the EWMA scheme and to $\theta = 1.02$ (resp. $\delta = 0.05$) because similar ones were obtained for the Shewhart scheme or most of the other values of θ (resp. δ) and

ϕ_1 and ϕ_2; however, tables 5 and 6 provide results for a wider constellation of parameters. Moreover, since the family of AR(2) processes includes the i.i.d. process and the sub-family of AR(1) processes: when $\phi_1 = \phi_2 = 0$, the estimates of the PMS of Type III (resp. Type IV) in tables 5 and 6 are close to the values of $\text{PMS}_{\text{III}}(\theta)$ (resp. the corresponding values of $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$) in Table 3 (resp. tables 5 and 6); when $\phi_2 = 0$, the values of $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ in Table 5 obviously coincide with the ones of $\text{PMS}_{\text{IV}}(\delta, \phi)$; finally, when $\phi_2 = 0$, the estimated results of the PMS of types III and IV in Table 5 are comparable to the ones we obtained for the AR(1) model in Table 4.

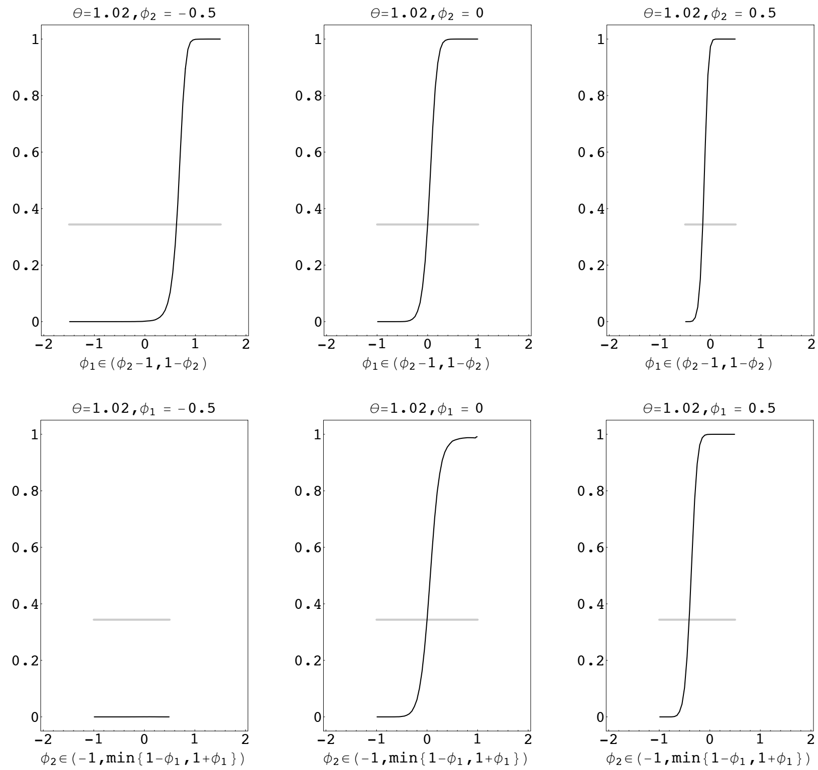

Figure 5: AR(2) model, EWMA — $\text{PMS}_{\text{III}}(\theta)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type III (traditional simultaneous scheme, black line), for $\phi_1 \in (\phi_2 - 1, 1 - \phi_2)$ [top] and $\phi_2 \in (-1, \min\{1 - \phi_1, 1 + \phi_1\})$ [bottom].

Secondly, when ϕ_2 takes a fixed value in $(-1, 1)$ such as $\phi_2 = -0.5, 0, 0.5$, the estimates of the PMS of Type III (resp. Type IV) increase (resp. decrease) with $\phi_1 \in (\phi_2 - 1, 1 - \phi_2)$ instead of being constant (resp. increasing), as shown by Figure 5 (resp. 6); analogously, when $\phi_1 = -0.5, 0, 0.5$, the estimates of the PMS of Type III (resp. Type IV) also increase (resp. tend to decrease) with $\phi_2 \in (-1, \min\{1 - \phi_1, 1 + \phi_1\})$ when they should not vary (resp. should increase). Curiously enough when $\phi_2 = -0.5$ (resp. $\phi_1 = -0.5$) and $\phi_1 \in (\phi_2 - 1, 0]$ (resp. $\phi_2 \in (-1, \min\{1 - \phi_1, 1 + \phi_1\})$) the estimates of the PMS of Type III are all very close to zero, as Figure 5 and Table 5 suggest, i.e., the individual EWMA chart for the process variance tends to signal earlier than the one for the process mean most of the time, when there is a small upward shift in σ^2. A comparable result was obtained for the estimates of the PMS of Type IV: when $\phi_2 = -0.5$ (resp. $\phi_1 = -0.5$) and $\phi_1 \in (\phi_2 - 1, 0]$ (resp. $\phi_2 \in (-1, \min\{1-\phi_1, 1+\phi_1\})$), these estimates are very close to 1 (see Figure 6 or tables 5 and 6), certainly because the individual EWMA chart for σ^2 tends to trigger alarms sooner than the one for μ most of the time, when there is a small shift in the process mean.

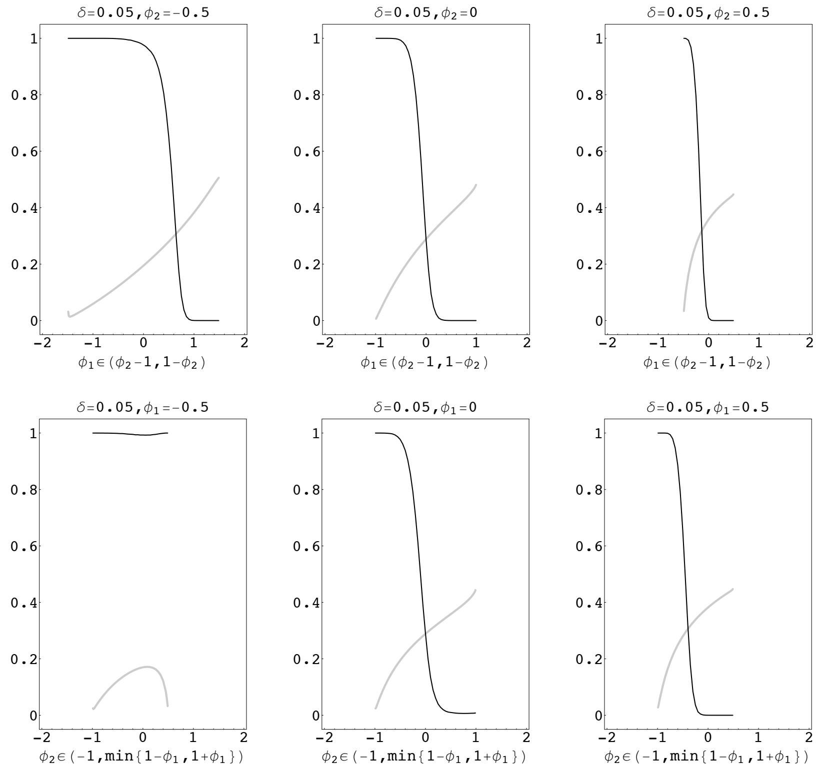

Figure 6: AR(2) model, EWMA — $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type IV (traditional simultaneous scheme, black line), for $\phi_1 \in (\phi_2 - 1, 1 - \phi_2)$ [top] and $\phi_2 \in (-1, \min\{1 - \phi_1, 1 + \phi_1\})$ [bottom].

Thirdly, we ought to refer that the discrepancies between the estimates of both PMS and their corresponding values are all too apparent not only in figures 5 and 6, but also in tables 5 and 6. In fact, if we (un)conscientiously disregard the autocorrelation structure of the output and adopt traditional simultaneous schemes for the process mean and variance instead of simultaneous residual schemes, we are bound to overestimate or underestimate the PMS depending on the values of the parameters ϕ_1 and ϕ_2. For example, for the simultaneous Shewhart residual scheme, we got:

Table 5: AR(2) model, $\phi_2 = -0.5, 0, 0.5$ — estimates of PMS of Type III of traditional simultaneous scheme; $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ of simultaneous residual scheme; estimates of PMS of Type IV of traditional simultaneous scheme.

		$\phi_2 = -0.5,\ \phi_1 \in (\phi_2 - 1, 1 - \phi_2)$											
		−1.45	−1.40	−0.90	−0.50	−0.30	0	0.30	0.50	0.90	1.40	1.45	λ
θ	1.02	0.000000	0.000000	0.000000	0.000000	0.000010	0.000720	0.011150	0.071770	0.842410	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000020	0.000080	0.001620	0.016380	0.104460	0.988900	1.000000	1.000000	0.05
	1.10	0.000000	0.000000	0.000000	0.000010	0.000110	0.001220	0.015700	0.085110	0.793730	0.999990	1.000000	1
		0.000000	0.000000	0.000000	0.000010	0.000030	0.001010	0.008140	0.044860	0.884560	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000020	0.000260	0.002620	0.024310	0.098430	0.738910	0.999950	1.000000	1
		0.000000	0.000000	0.000000	0.000010	0.000050	0.000540	0.004940	0.024110	0.564330	1.000000	1.000000	0.05
δ	0.05	0.138986	0.197873	0.393962	0.437392	0.450454	0.464818	0.475377	0.481078	0.490475	0.500429	0.501417	1
		0.013758	0.017160	0.070914	0.121760	0.149613	0.194614	0.243968	0.279727	0.360091	0.483677	0.496673	0.05
	0.50	0.000000	0.000001	0.007995	0.012968	0.017160	0.027299	0.045670	0.066361	0.154214	0.556766	0.630473	1
		0.131107	0.084971	0.001655	0.001145	0.001214	0.001594	0.002504	0.003731	0.011401	0.271185	0.510216	0.05
	1.00	0.000000	0.000000	0.000139	0.001325	0.001960	0.003169	0.005704	0.009563	0.041267	0.666765	0.811956	1
		0.000000	0.000000	0.002232	0.000810	0.000516	0.000404	0.000503	0.000768	0.004035	0.450430	0.775576	0.05
δ	0.05	1.000000	1.000000	1.000000	1.000000	0.999950	0.999360	0.988830	0.922900	0.139220	0.000000	0.000000	1
		1.000000	1.000000	0.999980	0.998540	0.994410	0.976350	0.891690	0.650750	0.004600	0.000000	0.000000	0.05
	0.50	1.000000	1.000000	0.999990	0.990560	0.959620	0.848090	0.600330	0.349410	0.033800	0.000000	0.000000	1
		0.871920	0.675190	0.034890	0.036680	0.031800	0.028900	0.023530	0.017040	0.000930	0.000000	0.000000	0.05
	1.00	0.943180	0.952440	0.518420	0.159430	0.104840	0.072870	0.051150	0.035160	0.006500	0.000000	0.000000	1
		0.418460	0.223430	0.005860	0.004660	0.004220	0.004350	0.003760	0.003240	0.000320	0.000000	0.000000	0.05

		$\phi_2 = 0,\ \phi_1 \in (\phi_2 - 1, 1 - \phi_2)$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
θ	1.02	0.000000	0.000000	0.000000	0.000030	0.007440	0.475730	0.937650	0.987950	0.999320	1.000000	1.000000	1
		0.000000	0.000000	0.000010	0.000240	0.009180	0.344010	0.986890	0.999840	1.000000	1.000000	1.000000	0.05
	1.10	0.000000	0.000000	0.000000	0.000190	0.011080	0.398280	0.894630	0.976030	0.997670	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000190	0.004460	0.099910	0.777120	0.995430	0.999990	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000680	0.017310	0.330990	0.827600	0.951130	0.993990	1.000000	1.000000	1
		0.000000	0.000000	0.000010	0.000110	0.002510	0.043050	0.367860	0.907220	0.999770	1.000000	1.000000	0.05
δ	0.05	0.245605	0.331756	0.429811	0.457404	0.470828	0.481977	0.488703	0.492034	0.494900	0.497702	0.498534	1
		0.021403	0.040098	0.109687	0.169903	0.221933	0.288653	0.346313	0.381892	0.417014	0.455573	0.467676	0.05
	0.50	0.000411	0.003322	0.009211	0.017601	0.030661	0.061967	0.113516	0.165244	0.240161	0.363509	0.414297	1
		0.004417	0.002008	0.000750	0.000930	0.001422	0.002907	0.006195	0.010799	0.020925	0.055933	0.085086	0.05
	1.00	0.000000	0.000000	0.000620	0.001398	0.002359	0.005824	0.016431	0.035020	0.081256	0.229454	0.323318	1
		0.002409	0.012133	0.000422	0.000158	0.000141	0.000262	0.000794	0.002010	0.006279	0.032854	0.064217	0.05
δ	0.05	1.000000	1.000000	1.000000	0.999930	0.992470	0.482494	0.048980	0.009800	0.000680	0.000000	0.000000	1
		1.000000	1.000000	0.999830	0.992790	0.920050	0.287590	0.003720	0.000020	0.000000	0.000000	0.000000	0.05
	0.50	0.999980	0.999960	0.998730	0.955350	0.580780	0.059860	0.011580	0.003230	0.000270	0.000000	0.000000	1
		0.179330	0.160950	0.090900	0.043180	0.017730	0.003100	0.000300	0.000020	0.000000	0.000000	0.000000	0.05
	1.00	0.789070	0.744350	0.413940	0.134390	0.036730	0.005790	0.002280	0.000990	0.000050	0.000000	0.000000	1
		0.061170	0.050720	0.021960	0.006970	0.002150	0.000270	0.000020	0.000010	0.000000	0.000000	0.000000	0.05

		$\phi_2 = 0.5,\ \phi_1 \in (\phi_2 - 1, 1 - \phi_2)$											
		−0.45	−0.40	−0.30	−0.20	−0.10	0	0.10	0.20	0.30	0.40	0.45	λ
θ	1.02	0.000000	0.000010	0.002740	0.077290	0.454160	0.866230	0.984990	0.999460	1.000000	1.000000	1.000000	1
		0.000000	0.000140	0.014690	0.150070	0.632900	0.973230	0.999810	1.000000	1.000000	1.000000	1.000000	0.05
	1.10	0.000000	0.000020	0.006050	0.096260	0.438920	0.819820	0.971280	0.997940	0.999980	1.000000	1.000000	1
		0.000000	0.000140	0.009360	0.071180	0.297150	0.774640	0.992040	0.999980	1.000000	1.000000	1.000000	0.05
	1.20	0.000000	0.000030	0.011040	0.121060	0.422070	0.761270	0.942220	0.993600	0.999860	1.000000	1.000000	1
		0.000000	0.000190	0.006330	0.039050	0.143380	0.419480	0.873390	0.999000	1.000000	1.000000	1.000000	0.05
δ	0.05	0.425908	0.455393	0.474771	0.482647	0.487085	0.490010	0.492133	0.493786	0.495153	0.496379	0.497022	1
		0.103984	0.163775	0.241873	0.292843	0.329739	0.358237	0.381300	0.400664	0.417437	0.432431	0.439677	0.05
	0.50	0.008753	0.016719	0.039676	0.068293	0.100647	0.135836	0.1735485	0.214001	0.258226	0.309794	0.343080	1
		0.000772	0.000920	0.001907	0.003438	0.005566	0.008450	0.012366	0.017820	0.025858	0.039377	0.051647	0.05
	1.00	0.000549	0.001379	0.003606	0.007808	0.014985	0.026349	0.043585	0.069296	0.108180	0.171820	0.225618	1
		0.000500	0.000181	0.000221	0.000428	0.000858	0.001675	0.003188	0.006037	0.011775	0.025701	0.043189	0.05
δ	0.05	1.000000	0.999990	0.996360	0.919740	0.528740	0.116840	0.011320	0.000460	0.000000	0.000000	0.000000	1
		0.999520	0.993190	0.907280	0.612750	0.171630	0.009410	0.000100	0.000000	0.000000	0.000000	0.000000	0.05
	0.50	0.999360	0.989080	0.814290	0.413890	0.135410	0.030830	0.003830	0.000210	0.000000	0.000000	0.000000	1
		0.073170	0.069410	0.050980	0.025950	0.008120	0.001250	0.000100	0.000000	0.000000	0.000000	0.000000	0.05
	1.00	0.494000	0.286850	0.121670	0.054140	0.021170	0.005860	0.001270	0.000100	0.000000	0.000000	0.000000	1
		0.025000	0.021770	0.014870	0.006890	0.002250	0.000370	0.000010	0.000000	0.000000	0.000000	0.000000	0.05

Table 6: AR(2) model, $\phi_1 = -0.5, 0, 0.5$ — estimates of PMS of Type III of traditional simultaneous scheme; $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ of simultaneous residual scheme; estimates of PMS of Type IV of traditional simultaneous scheme.

		$\phi_1 = -0.5,\ \phi_2 \in (-1, \min\{1-\phi_1, 1+\phi_1\})$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.10	0.20	0.30	0.40	0.45	λ
	1.02	0.000000	0.000000	0.000000	0.000000	0.000010	0.000100	0.000040	0.000030	0.000000	0.000000	0.000000	1
		0.000000	0.000000	0.000000	0.000000	0.000040	0.000280	0.000330	0.000230	0.000050	0.000000	0.000000	0.05
θ	1.10	0.000000	0.000000	0.000000	0.000000	0.000020	0.000170	0.000160	0.000040	0.000010	0.000000	0.000000	1
		0.000000	0.000000	0.000000	0.000010	0.000040	0.000180	0.000290	0.000190	0.000080	0.000020	0.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000020	0.000140	0.000530	0.000590	0.000300	0.000040	0.000000	0.000000	1
		0.000000	0.000000	0.000000	0.000000	0.000040	0.000130	0.000190	0.000120	0.000080	0.000000	0.000000	0.05
	0.05	0.265973	0.331442	0.412628	0.437392	0.449385	0.457404	0.457915	0.456777	0.452369	0.437529	0.412047	1
		0.028191	0.041403	0.087209	0.121760	0.147475	0.169903	0.171484	0.167913	0.155284	0.122515	0.087187	0.05
δ	0.50	0.001453	0.012770	0.012890	0.012968	0.014656	0.017601	0.017971	0.017452	0.015354	0.010649	0.007313	1
		0.087440	0.032186	0.002498	0.001145	0.000918	0.000930	0.000947	0.000942	0.000887	0.000765	0.000832	0.05
	1.00	0.000000	0.000000	0.000784	0.001325	0.001320	0.001398	0.001437	0.001430	0.001289	0.000815	0.000293	1
		0.002662	0.238867	0.004256	0.000810	0.000272	0.000158	0.000160	0.000173	0.000204	0.000327	0.000685	0.05
	0.05	1.000000	1.000000	1.000000	1.000000	0.999960	0.999960	0.999990	1.000000	1.000000	1.000000	1.000000	1
		0.999970	0.999980	0.999700	0.998780	0.996370	0.993010	0.992950	0.994340	0.996680	0.999220	0.999960	0.05
δ	0.50	0.999850	0.999730	0.998040	0.990070	0.969240	0.954960	0.962810	0.976340	0.988620	0.998010	0.999780	1
		0.064170	0.061210	0.045270	0.035980	0.030520	0.045120	0.055290	0.071140	0.085320	0.090880	0.087500	0.05
	1.00	0.522970	0.457320	0.270600	0.161080	0.108920	0.137290	0.170260	0.219500	0.293710	0.422840	0.568690	1
		0.013980	0.013680	0.008090	0.004810	0.003540	0.007430	0.011440	0.017860	0.024520	0.029830	0.028010	0.05

		$\phi_1 = 0,\ \phi_2 \in (-1, \min\{1-\phi_1, 1+\phi_1\})$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
	1.02	0.000000	0.000000	0.000010	0.000580	0.021740	0.475280	0.829100	0.867240	0.874730	0.874670	0.875560	1
		0.000000	0.000000	0.000050	0.001520	0.020380	0.344220	0.907420	0.975770	0.986270	0.987750	0.987190	0.05
θ	1.10	0.000000	0.000000	0.000010	0.001230	0.028860	0.398150	0.762310	0.820770	0.837850	0.844030	0.842710	1
		0.000000	0.000000	0.000050	0.001080	0.009290	0.099020	0.484720	0.777010	0.902690	0.940200	0.945050	0.05
	1.20	0.000000	0.000000	0.000050	0.002830	0.034870	0.330540	0.679990	0.763560	0.793910	0.803910	0.804720	1
		0.000000	0.000000	0.000020	0.000550	0.004910	0.041310	0.197860	0.415300	0.653040	0.794970	0.818310	0.05
	0.05	0.325587	0.384272	0.446882	0.464818	0.473945	0.481977	0.487213	0.49001	0.492604	0.495445	0.496394	1
		0.040491	0.064109	0.139835	0.194614	0.237379	0.288653	0.331564	0.358237	0.385490	0.418047	0.429451	0.05
δ	0.50	0.015391	0.022880	0.021202	0.027299	0.037796	0.061967	0.099469	0.135836	0.188234	0.278600	0.319136	1
		0.072186	0.016166	0.002189	0.001594	0.001818	0.002907	0.005309	0.008450	0.014691	0.032342	0.044485	0.05
	1.00	0.000000	0.000113	0.003602	0.003169	0.003391	0.005824	0.013624	0.026349	0.055378	0.141787	0.197402	1
		0.326726	0.062125	0.002158	0.000404	0.000212	0.000262	0.000693	0.001675	0.004892	0.021400	0.037551	0.05
	0.05	1.000000	1.000000	0.999990	0.999160	0.975660	0.483339	0.147560	0.116520	0.108380	0.108150	0.108120	1
		1.000000	0.999960	0.998380	0.977150	0.856280	0.290070	0.027930	0.009950	0.006680	0.007060	0.007340	0.05
δ	0.50	0.999900	0.999640	0.987880	0.847910	0.412620	0.061730	0.027550	0.030070	0.036960	0.045330	0.047820	1
		0.097810	0.089420	0.053190	0.030550	0.012830	0.002850	0.001220	0.001230	0.001760	0.003140	0.003420	0.05
	1.00	0.554140	0.467040	0.197720	0.073030	0.024560	0.005800	0.004050	0.006100	0.008850	0.012780	0.013620	1
		0.022550	0.018790	0.009510	0.004170	0.001590	0.000290	0.000220	0.000340	0.000730	0.001490	0.001790	0.05

		$\phi_1 = 0.5,\ \phi_2 \in (-1, \min\{1-\phi_1, 1+\phi_1\})$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.10	0.20	0.30	0.40	0.45	λ
	1.02	0.000000	0.000000	0.000270	0.073120	0.633280	0.988150	0.997440	0.999790	0.999980	1.000000	1.000000	1
		0.000000	0.000000	0.001210	0.103060	0.768940	0.999810	1.000000	1.000000	1.000000	1.000000	1.000000	0.05
θ	1.10	0.000000	0.000000	0.000760	0.086300	0.576680	0.975240	0.994340	0.999220	0.999970	1.000000	1.000000	1
		0.000000	0.000000	0.000990	0.043900	0.359030	0.994990	0.999880	1.000000	1.000000	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.001740	0.097770	0.515500	0.951820	0.985120	0.997400	0.999900	1.000000	1.000000	1
		0.000000	0.000000	0.000700	0.023350	0.158150	0.907670	1.000000	1.000000	1.000000	1.000000	1.000000	0.05
	0.05	0.379857	0.426024	0.469467	0.481078	0.486905	0.492034	0.493287	0.494409	0.495447	0.496467	0.497046	1
		0.061691	0.101246	0.211369	0.279727	0.328298	0.381892	0.396572	0.410115	0.422683	0.434450	0.440385	0.05
δ	0.50	0.035274	0.034565	0.043821	0.066361	0.098203	0.165244	0.193828	0.226430	0.264326	0.311321	0.343262	1
		0.041673	0.011949	0.003422	0.003731	0.005337	0.010799	0.014136	0.018945	0.026321	0.039280	0.051442	0.05
	1.00	0.000045	0.005096	0.009032	0.009563	0.014028	0.035020	0.049987	0.072778	0.108633	0.170302	0.224296	1
		0.209786	0.038736	0.002032	0.000768	0.000771	0.002010	0.003244	0.005635	0.010750	0.024221	0.042037	0.05
	0.05	1.000000	1.000000	0.999740	0.923440	0.340790	0.009680	0.001720	0.000120	0.000000	0.000000	0.000000	1
		1.000000	1.000000	0.979870	0.653430	0.084830	0.000100	0.000000	0.000000	0.000000	0.000000	0.000000	0.05
δ	0.50	1.000000	1.000000	0.921130	0.352000	0.070190	0.003450	0.000840	0.000080	0.000000	0.000000	0.000000	1
		0.078310	0.072090	0.042240	0.016830	0.003880	0.000060	0.000000	0.000000	0.000000	0.000000	0.000000	0.05
	1.00	0.995080	0.821420	0.127650	0.035080	0.010130	0.000970	0.000150	0.000000	0.000000	0.000000	0.000000	1
		0.017940	0.016050	0.007670	0.002640	0.000800	0.000060	0.000000	0.000000	0.000000	0.000000	0.000000	0.05

- $\mathrm{PMS}_{\mathrm{III}}(1.1) = 0.397714$, while the estimates can take values from 0.000000 ($\phi_1 = -0.9, \phi_2 = -0.5$) to 1.000000 ($\phi_1 = 0.5, \phi_2 = 0.45$), but also values in between, such as 0.438920 ($\phi_1 = -0.1, \phi_2 = 0.5$) and 0.011080 ($\phi_1 = -0.3, \phi_2 = 0$);
- $\mathrm{PMS}_{\mathrm{IV}}(0.5, -0.5, -0.5) = 0.012968$, $\mathrm{PMS}_{\mathrm{IV}}(0.5, 1.45, -0.5) = 0.630473$, $\mathrm{PMS}_{\mathrm{IV}}(0.5, -0.1, 0.5) = 0.100647$ and $\mathrm{PMS}_{\mathrm{IV}}(0.5, 0, 0.5) = 0.135836$, nevertheless, the estimated PMS are equal to 0.990070, 0.000000, 0.135410 and 0.030070, respectively.

To sum up, these results and the ones in tables 5 and 6 are in accordance to the ones we reported in the previous subsection for the AR(1) model — when we fail to recognize an AR(2) process and mistakenly design a simultaneous scheme assuming i.i.d. output, the estimates of the PMS of Type III (resp. Type IV) tend to increase (resp. decrease) with parameters ϕ_1 and ϕ_2. As a consequence, only simultaneous residual schemes will give protection to both types of MS.

4.3. ARMA(1,1) model

Autocorrelated output from stable (continuous) processes frequently follow ARMA models of low order ([13, p. 2]), such as ARMA(1,1). The process $\{Y_{i,j}\}$ follows a stationary and invertible ARMA(1,1) model with mean μ_0, variance $\sigma_0^2 = \gamma_0$, autoregressive parameter ϕ and moving average parameter α, for every i, if

(4.7) $$Y_{i,j} = \mu_0 + \phi\,(Y_{i,j-1} - \mu_0) + \varepsilon_{i,j} - \alpha\,\varepsilon_{i,j-1} ,$$

where $-1 < \phi, \alpha < 1$ and $\varepsilon_{i,j} \sim_{i.i.d.} \mathcal{N}(0, \sigma_\varepsilon^2)$, with $\sigma_\varepsilon^2 = \frac{1-\phi^2}{1+\alpha^2-2\phi\alpha} \times \sigma_0^2$.

Now it is time to investigate the impact of falsely assuming i.i.d. output — rather than recognizing the ARMA(1,1) nature of the output — in the PMS of both types III and IV.

Once again we restricted ourselves to the EWMA scheme, $\theta = 1.02$ and $\delta = 0.05$ when it comes to graphical illustrations because the graphs we obtained for the Shewhart scheme or most of the other values of θ, δ, ϕ and α are similar to the ones in figures 7 and 8; tables 7 and 8 provide complementary results. In addition to this, we should remind the reader that the sub-family of AR(1) processes and the i.i.d. process are particular cases of the ARMA(1,1) processes. As a consequence we were able to check the values we got for $\mathrm{PMS}_{\mathrm{IV}}(\delta, \phi, \alpha)$ in Table 7 (resp. Table 8) when $\alpha = 0$ (resp. $\phi = \alpha = 0$), with the ones in Table 4. Unsurprisingly, the estimates of the PMS of Type III (resp. Type IV) in tables 7 and 8 are close to the values of $\mathrm{PMS}_{\mathrm{III}}(\theta)$ (resp. $\mathrm{PMS}_{\mathrm{IV}}(\delta, \phi, \alpha)$) in Table 3 (resp. tables 7 and 8); moreover, when $\alpha = 0$, the estimates of the PMS of types III and IV in Table 7 are comparable to the ones we obtained for the AR(1) model in Table 4.

Tables 7 and 8 and Figure 8 lead us to state that $\text{PMS}_{\text{IV}}(\delta, \phi, \alpha)$ seems to: decrease with α, for varying or fixed ϕ, unlike $\text{PMS}_{\text{IV}}(\delta, \phi)$ and $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ that tend to increase with the model parameter(s); increase with the autoregressive parameter ϕ like in the two previous models. Once again the adoption of a simultaneous EWMA residual scheme in place of a simultaneous Shewhart residual scheme yields a decrease of the $\text{PMS}_{\text{IV}}(\delta, \phi, \alpha)$, for most values of δ, ϕ and α of this specific output process.

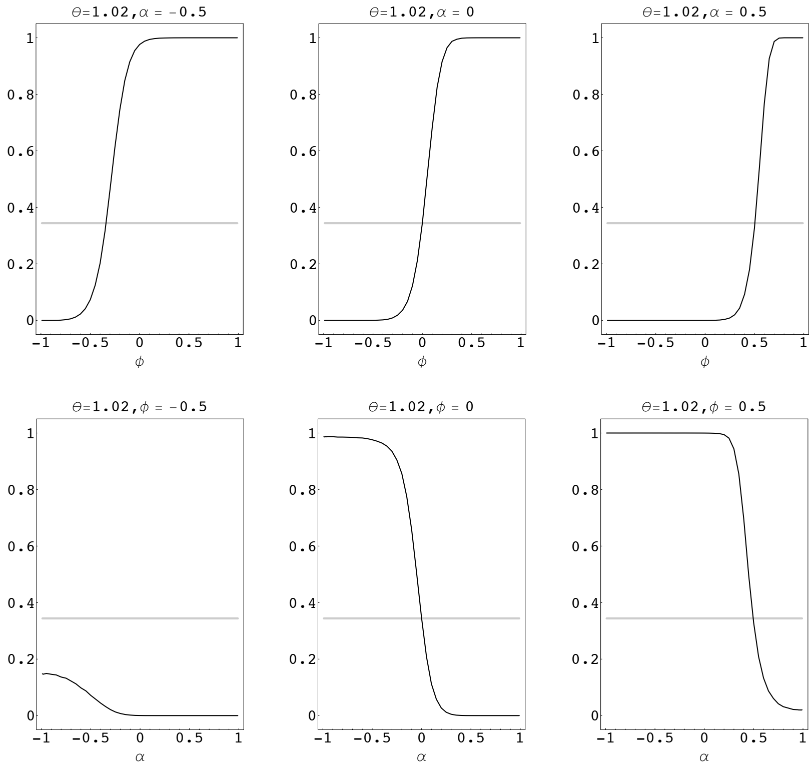

Figure 7: ARMA(1,1) model, EWMA — $\text{PMS}_{\text{III}}(\theta)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type III (traditional simultaneous scheme, black line), for $\phi \in (-1, 1)$ [top] and $\alpha \in (-1, 1)$ [bottom].

When we ignore that the output follows an ARMA$(1, 1)$ model, the estimates of the PMS of Type III increase (resp. decrease) from 0 to 1 with ϕ (resp. α), although $\text{PMS}_{\text{III}}(\theta)$ is constant when we adopt simultaneous residual schemes, as depicted by figures 7 and 8; curiously, when $\phi = -0.5$ the estimates of the PMS of Type III estimates are in general smaller than $\text{PMS}(\theta)$ in the EWMA case for $\theta = 1.02$, as shown in Figure 7, but also for the Shewhart case and $\theta = 1.1, 1.2$. Furthermore, Figure 8 suggests that the estimates of PMS of Type IV decrease (resp. increase) with ϕ (resp. α); however, the values of $\text{PMS}_{\text{IV}}(\delta, \phi, \alpha)$ tend to increase (resp. tend to decrease) with ϕ (resp. α) for fixed α (resp. ϕ), as portrayed by Table 7 (resp. 8). It goes without saying that correlation has quite an impact on the PMS of types III and IV. For example, for the simultaneous EWMA residual scheme, we got:

- $\mathrm{PMS_{III}}(1.2) = 0.042865$, whereas the estimates take values from 0.000000 ($\phi = -0.5, \alpha = 0.5$) to 1.000000 ($\phi = 0.9, \alpha = 0.5$) and values in the interval $(0, 1)$, such as 0.371910 $\phi = 0, \alpha = -0.5$) and 0.992420 ($\phi = 0.5, \alpha = -0.3$);
- $\mathrm{PMS_{IV}}(1.0, -0.5, 0.6) = 0.045997$, $\mathrm{PMS_{IV}}(1.0, 0.9, 0.5) = 0.0121289$, $\mathrm{PMS_{IV}}(1.0, 0, -0.6) = 0.002448$ and $\mathrm{PMS_{IV}}(1.0, 0.5, -0.3) = 0.007234$, however, the estimated PMS are equal to 0.013120, 0.000000, 0.000180 and 0.000000, respectively.

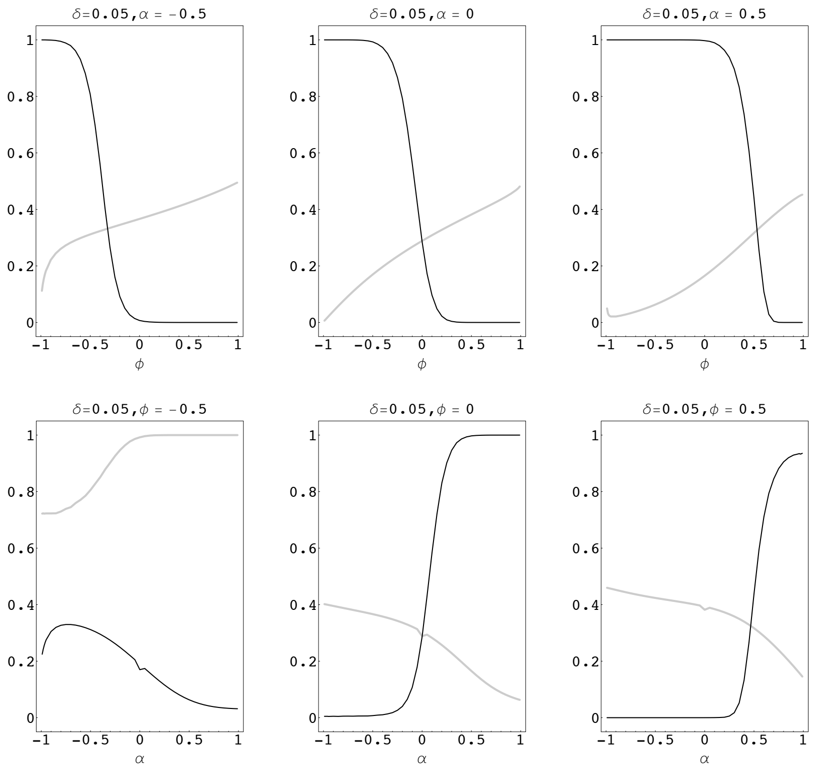

Figure 8: ARMA(1,1) model, EWMA — $\mathrm{PMS_{IV}}(\delta, \phi, \alpha)$ (simultaneous residual scheme, grey line) and estimates of PMS of Type IV (traditional simultaneous scheme, black line), for $\phi \in (-1, 1)$ [top] and $\alpha \in (-1, 1)$ [bottom].

Once again, the numerical results in tables 7 and 8 revealed that substituting the traditional Shewhart by a traditional EWMA chart can be frequently followed by an increase of the estimates of the PMS of types III and IV, even though both $\mathrm{PMS_{III}}(\theta)$ and $\mathrm{PMS_{IV}}(\delta, \phi, \alpha)$ decrease (in general) when a simultaneous EWMA residual scheme replaces a simultaneous Shewhart residual scheme.

Table 7: ARMA(1,1) model, $\alpha = -0.5, 0, 0.5$ — estimates of PMS of Type III of traditional simultaneous scheme; $\mathrm{PMS_{IV}}(\delta,\phi,\alpha)$ of simultaneous residual scheme; estimates of PMS of Type IV of traditional simultaneous scheme.

		$\alpha=-0.5,\ \phi\in(-1,1)$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
θ	1.02	0.000000	0.000020	0.014870	0.237770	0.663090	0.897370	0.975290	0.994530	0.999690	1.000000	1.000000	1
		0.000000	0.000060	0.005400	0.073120	0.462260	0.976270	0.999600	0.999990	1.000000	1.000000	1.000000	0.05
	1.10	0.000000	0.000090	0.021090	0.218110	0.578860	0.846470	0.956710	0.988500	0.999110	1.000000	1.000000	1
		0.000000	0.000090	0.003500	0.033410	0.165390	0.749910	0.993510	0.999860	1.000000	1.000000	1.000000	0.05
	1.20	0.000000	0.000250	0.030950	0.206270	0.490990	0.777930	0.926250	0.976410	0.997130	1.000000	1.000000	1
		0.000000	0.000010	0.002650	0.018190	0.077920	0.371910	0.911080	0.996570	0.999960	1.000000	1.000000	0.05
δ	0.05	0.461709	0.470959	0.481183	0.484953	0.487537	0.490736	0.493693	0.495669	0.497736	0.500100	0.500867	1
		0.181982	0.221266	0.282392	0.311712	0.334569	0.366417	0.399620	0.423733	0.450097	0.479688	0.488022	0.05
	0.50	0.028599	0.035284	0.059992	0.080718	0.102717	0.145310	0.211128	0.278578	0.378206	0.532869	0.588638	1
		0.002089	0.001917	0.002903	0.004068	0.005543	0.009165	0.017346	0.030600	0.066607	0.223590	0.346540	0.05
	1.00	0.004555	0.003995	0.006088	0.009456	0.014463	0.028959	0.066950	0.131011	0.281473	0.614743	0.731917	1
		0.001019	0.000378	0.000313	0.000464	0.000743	0.001773	0.005713	0.016133	0.062199	0.362196	0.581421	0.05
δ	0.05	1.000000	0.999870	0.984940	0.742730	0.299680	0.085390	0.019830	0.004300	0.000260	0.000000	0.000000	1
		0.999830	0.999230	0.980050	0.807800	0.266820	0.006970	0.000160	0.000030	0.000000	0.000000	0.000000	0.05
	0.50	0.995940	0.978720	0.702140	0.190550	0.054190	0.020570	0.006460	0.001870	0.000120	0.000000	0.000000	1
		0.317100	0.337220	0.151560	0.028600	0.005020	0.000720	0.000090	0.000000	0.000000	0.000000	0.000000	0.05
	1.00	0.517520	0.399720	0.116810	0.021890	0.007260	0.003600	0.001280	0.000500	0.000020	0.000000	0.000000	1
		0.109900	0.105250	0.027830	0.003170	0.000570	0.000210	0.000050	0.000000	0.000000	0.000000	0.000000	0.05

		$\alpha=0,\ \phi\in(-1,1)$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
θ	1.02	0.000000	0.000000	0.000000	0.000070	0.007220	0.474190	0.938290	0.988060	0.999320	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000210	0.009000	0.344310	0.987430	0.999840	1.000000	1.000000	1.000000	0.05
	1.10	0.000000	0.000000	0.000000	0.000210	0.011400	0.398590	0.895530	0.975530	0.997850	1.000000	1.000000	1
		0.000000	0.000000	0.000000	0.000210	0.004680	0.100180	0.779820	0.995310	1.000000	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000630	0.016890	0.331170	0.828060	0.950540	0.994370	0.999990	1.000000	1
		0.000000	0.000000	0.000000	0.000070	0.002620	0.041820	0.366710	0.909390	0.999690	1.000000	1.000000	0.05
δ	0.05	0.245605	0.331756	0.429811	0.457404	0.470828	0.481977	0.488703	0.492034	0.494900	0.497702	0.498534	1
		0.021403	0.040098	0.109687	0.169903	0.221933	0.288653	0.346313	0.381892	0.417014	0.455573	0.467676	0.05
	0.50	0.000411	0.003322	0.009211	0.017601	0.030661	0.061967	0.113516	0.165244	0.240161	0.363509	0.414297	1
		0.004417	0.002008	0.000750	0.000930	0.001422	0.002907	0.006195	0.010799	0.020925	0.055933	0.085086	0.05
	1.00	0.000000	0.000000	0.000620	0.001398	0.002359	0.005824	0.016431	0.035020	0.081256	0.229454	0.323318	1
		0.002409	0.012133	0.000422	0.000158	0.000141	0.000262	0.000794	0.002010	0.006279	0.032854	0.064217	0.05
δ	0.05	1.000000	1.000000	1.000000	0.999960	0.991760	0.484249	0.051580	0.009020	0.000510	0.000000	0.000000	1
		1.000000	1.000000	0.999770	0.992870	0.918780	0.286200	0.003850	0.000050	0.000000	0.000000	0.000000	0.05
	0.50	1.000000	0.999970	0.998620	0.954720	0.578830	0.062320	0.010940	0.003100	0.000200	0.000000	0.000000	1
		0.178620	0.159300	0.091340	0.043710	0.017720	0.003220	0.000380	0.000040	0.000000	0.000000	0.000000	0.05
	1.00	0.789230	0.746060	0.414820	0.134970	0.036230	0.005680	0.001970	0.000800	0.000110	0.000000	0.000000	1
		0.061530	0.048890	0.021810	0.007440	0.002110	0.000300	0.000130	0.000000	0.000000	0.000000	0.000000	0.05

		$\alpha=0.5,\ \phi\in(-1,1)$											
		−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
θ	1.02	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.047350	0.695910	0.983310	0.999980	1.000000	1
		0.000000	0.000000	0.000000	0.000000	0.000000	0.000040	0.019940	0.326950	0.986850	1.000000	1.000000	0.05
	1.10	0.000000	0.000000	0.000000	0.000000	0.000000	0.000070	0.051120	0.599140	0.965840	0.999960	1.000000	1
		0.000000	0.000000	0.000000	0.000000	0.000000	0.000060	0.009000	0.130160	0.847100	1.000000	1.000000	0.05
	1.20	0.000000	0.000000	0.000000	0.000000	0.000000	0.000120	0.056460	0.506320	0.928280	0.999870	1.000000	1
		0.000000	0.000000	0.000000	0.000000	0.000000	0.000010	0.005150	0.065750	0.531280	1.000000	1.000000	0.05
δ	0.05	0.179105	0.215087	0.322009	0.382990	0.421192	0.456249	0.476720	0.485569	0.491830	0.496163	0.496981	1
		0.021150	0.021172	0.038021	0.063325	0.097070	0.165267	0.252675	0.316783	0.379364	0.433895	0.445274	0.05
	0.50	0.000000	0.000017	0.007394	0.010726	0.012774	0.021023	0.046081	0.085810	0.161890	0.287884	0.325411	1
		0.381255	0.220021	0.015474	0.003924	0.001904	0.001426	0.002305	0.004426	0.010519	0.030728	0.041320	0.05
	1.00	0.000000	0.000000	0.000000	0.000077	0.001070	0.002618	0.004777	0.010706	0.034000	0.126378	0.172331	1
		0.000000	0.000000	0.126200	0.009655	0.002860	0.000553	0.000314	0.000545	0.001976	0.012129	0.019974	0.05
δ	0.05	1.000000	1.000000	1.000000	1.000000	1.000000	0.999990	0.948060	0.266680	0.010810	0.000000	0.000000	1
		1.000000	1.000000	1.000000	1.000000	0.999980	0.997240	0.896750	0.440600	0.004400	0.000000	0.000000	0.05
	0.50	1.000000	1.000000	1.000000	1.000000	0.999980	0.973430	0.327820	0.053930	0.005320	0.000000	0.000000	1
		0.162440	0.137210	0.083930	0.062050	0.046100	0.031970	0.020810	0.009870	0.000580	0.000000	0.000000	0.05
	1.00	0.850880	0.877470	0.892810	0.772690	0.476960	0.114260	0.022080	0.008290	0.001590	0.000000	0.000000	1
		0.054780	0.044580	0.022150	0.012550	0.007630	0.003710	0.001600	0.000880	0.000090	0.000000	0.000000	0.05

Table 8: ARMA(1,1) model, $\phi = -0.5, 0, 0.5$ — estimates of PMS of Type III of traditional simultaneous scheme; $\mathrm{PMS_{IV}}(\delta, \phi, \alpha)$ of simultaneous residual scheme; estimates of PMS of Type IV of traditional simultaneous scheme.

	$\phi = -0.5,\ \alpha \in (-1,1)$											
	−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
1.02	0.457660	0.452470	0.400090	0.238130	0.039670	0.000050	0.000000	0.000000	0.000000	0.000000	0.000000	1
	0.149230	0.146220	0.122350	0.071460	0.021110	0.000270	0.000000	0.000000	0.000000	0.000000	0.000000	0.05
θ 1.10	0.408010	0.406120	0.357100	0.216580	0.046720	0.000220	0.000000	0.000000	0.000000	0.000000	0.000000	1
	0.071460	0.072440	0.058790	0.034090	0.010020	0.000250	0.000000	0.000000	0.000000	0.000000	0.000000	0.05
1.20	0.359950	0.358600	0.318480	0.200150	0.055320	0.000480	0.000000	0.000000	0.000000	0.000000	0.000000	1
	0.042440	0.041090	0.032110	0.018770	0.005870	0.000190	0.000000	0.000000	0.000000	0.000000	0.000000	0.05
0.05	0.480395	0.484236	0.486991	0.484953	0.479980	0.457404	0.425647	0.382990	0.333826	0.300133	0.295870	1
	0.275450	0.305104	0.329699	0.311712	0.273922	0.169903	0.103053	0.063325	0.041691	0.033141	0.032201	0.05
δ 0.50	0.061584	0.078360	0.096743	0.080718	0.055258	0.017601	0.012169	0.010726	0.009900	0.006023	0.005301	1
	0.003321	0.004088	0.005077	0.004068	0.002669	0.000930	0.001503	0.003924	0.016522	0.036310	0.036225	0.05
1.00	0.008038	0.009944	0.012695	0.009456	0.005499	0.001398	0.001089	0.000077	0.000000	0.000000	0.000000	1
	0.000588	0.000564	0.000622	0.000464	0.000294	0.000158	0.001836	0.009655	0.138196	0.128850	0.106785	0.05
0.05	0.511620	0.516940	0.573440	0.745730	0.958320	0.999910	1.000000	1.000000	1.000000	1.000000	1.000000	1
	0.722820	0.722860	0.745010	0.805760	0.902680	0.992200	1.000000	1.000000	1.000000	1.000000	1.000000	0.05
δ 0.50	0.140790	0.138420	0.144850	0.189830	0.416760	0.955190	0.999800	1.000000	1.000000	1.000000	1.000000	1
	0.033610	0.035080	0.030420	0.028340	0.030000	0.044150	0.058530	0.060210	0.063270	0.063950	0.065000	0.05
1.00	0.022700	0.021930	0.020800	0.021210	0.036800	0.137390	0.487070	0.772930	0.905330	0.941310	0.943090	1
	0.004240	0.004660	0.003900	0.002900	0.003760	0.007430	0.011170	0.013190	0.014090	0.014060	0.013580	0.05

	$\phi = 0,\ \alpha \in (-1,1)$											
	−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
1.02	0.908920	0.910160	0.906110	0.898620	0.868240	0.473770	0.003420	0.000020	0.000000	0.000000	0.000000	1
	0.987130	0.986940	0.984640	0.976120	0.935300	0.345880	0.004750	0.000040	0.000000	0.000000	0.000000	0.05
θ 1.10	0.864100	0.864520	0.859050	0.846670	0.801580	0.398260	0.006000	0.000040	0.000000	0.000000	0.000000	1
	0.849400	0.847540	0.825910	0.752340	0.548550	0.100580	0.002220	0.000000	0.000000	0.000000	0.000000	0.05
1.20	0.801860	0.802200	0.799270	0.775030	0.712870	0.333620	0.010280	0.000130	0.000000	0.000000	0.000000	1
	0.499590	0.495990	0.463250	0.371320	0.224130	0.042200	0.001450	0.000040	0.000000	0.000000	0.000000	0.05
0.05	0.494277	0.493794	0.492182	0.490736	0.488951	0.481977	0.472243	0.456249	0.430660	0.397957	0.389504	1
	0.399002	0.395418	0.381231	0.366417	0.348203	0.288653	0.228148	0.165267	0.109878	0.073722	0.067526	0.05
δ 0.50	0.253890	0.232569	0.176869	0.145310	0.118661	0.061967	0.035647	0.021023	0.015768	0.014755	0.014389	1
	0.029515	0.023790	0.013012	0.009165	0.006755	0.002907	0.001799	0.001426	0.002150	0.004869	0.005802	0.05
1.00	0.130325	0.101906	0.047069	0.028959	0.018979	0.005824	0.003498	0.002618	0.001933	0.000368	0.000175	1
	0.024256	0.015134	0.003745	0.001773	0.001023	0.000262	0.000262	0.000553	0.003228	0.010105	0.014359	0.05
0.05	0.076440	0.077440	0.079140	0.085810	0.113210	0.480624	0.995560	1.000000	1.000000	1.000000	1.000000	1
	0.004430	0.005020	0.005360	0.007330	0.017850	0.286010	0.946410	0.997210	0.999960	0.999980	0.999970	0.05
δ 0.50	0.021250	0.021240	0.020560	0.021970	0.021780	0.062320	0.635820	0.972370	0.999010	0.999890	0.999860	1
	0.000840	0.000870	0.000550	0.000780	0.000890	0.003330	0.017170	0.032600	0.043610	0.046410	0.047700	0.05
1.00	0.004280	0.004370	0.004510	0.003810	0.003410	0.005260	0.035820	0.114660	0.277980	0.425890	0.440990	1
	0.000230	0.000260	0.000160	0.000200	0.000150	0.000220	0.002000	0.003800	0.005520	0.006120	0.005910	0.05

	$\phi = 0.5,\ \alpha \in (-1,1)$											
	−0.95	−0.90	−0.70	−0.50	−0.30	0	0.30	0.50	0.70	0.90	0.95	λ
1.02	0.995020	0.994880	0.994800	0.994460	0.993490	0.988250	0.936280	0.693570	0.272970	0.118170	0.107720	1
	0.999980	0.999980	1.000000	0.999980	0.999960	0.999820	0.943680	0.328860	0.060820	0.021760	0.020650	0.05
θ 1.10	0.990200	0.989810	0.989520	0.988890	0.986900	0.975600	0.889050	0.601510	0.238770	0.110700	0.104630	1
	0.999870	0.999850	0.999820	0.999870	0.999590	0.995420	0.600210	0.131810	0.029260	0.011710	0.010760	0.05
1.20	0.979300	0.978080	0.978060	0.976880	0.972870	0.952220	0.813460	0.506270	0.207750	0.106540	0.099790	1
	0.997670	0.997770	0.997420	0.996350	0.992420	0.908190	0.278260	0.064790	0.015310	0.006500	0.005850	0.05
0.05	0.499689	0.499118	0.497066	0.495669	0.494662	0.492034	0.489918	0.485569	0.477396	0.461660	0.455367	1
	0.456381	0.452179	0.436525	0.423733	0.412621	0.381892	0.358381	0.316783	0.256556	0.182519	0.162304	0.05
δ 0.50	0.517509	0.482040	0.354280	0.278578	0.236756	0.165244	0.130423	0.085810	0.048758	0.025838	0.021982	1
	0.195365	0.154296	0.058931	0.030600	0.021007	0.010799	0.007627	0.004426	0.002485	0.001683	0.001633	0.05
1.00	0.601221	0.528941	0.258189	0.131011	0.083377	0.035020	0.022134	0.010706	0.005346	0.003420	0.003090	1
	0.399714	0.300459	0.063075	0.016133	0.007234	0.002010	0.001179	0.000545	0.000364	0.000598	0.000806	0.05
0.05	0.003560	0.003520	0.003890	0.003850	0.004640	0.009530	0.050440	0.268780	0.694960	0.865420	0.875450	1
	0.000020**	0.000020	0.000000	0.000020	0.000010	0.000070	0.000850	0.009530	0.037110	0.058900	0.060270	0.05
δ 0.50	0.001580	0.001420	0.001600	0.001820	0.002090	0.003150	0.013320	0.053700	0.171200	0.282310	0.295480	1
	0.000020*	0.000020*	0.000000*	0.000020*	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0.05
1.00	0.000400	0.000440	0.000610	0.000520	0.000580	0.000860	0.002340	0.007770	0.019470	0.030020	0.031220	1
	0.000030	0.000000	0.000000	0.000000	0.000000	0.000010	0.000160	0.000930	0.003890	0.006170	0.006440	0.05

5. CONCLUDING REMARKS

The introduction of automatic measuring devices and the subsequent increase in the frequency of the measurements led to autocorrelated output, a major issue in the process industries, as [13, p. iii] felt bound to point out.

This paper confirms that autocorrelation can cause traditional simultaneous control schemes to produce misleading signals either more or less frequently than simultaneous residual schemes, depending on the type of autocorrelation. In fact, if we ignore or neglect the autocorrelation structure of the output then we can obtain estimates of PMS of types III and IV smaller than the ones we would obtain if we adopted simultaneous residual schemes to monitor the mean and variance of AR(1), AR(2) and ARMA(1,1) processes; furthermore, the regions where these schemes are superseded by the traditional ones tend to be symmetric for the PMS of types III and IV, thus, only simultaneous residual schemes will give the necessary protection to MS of both types, as previously mentioned by [8] for the AR(1) process. Some monotonicity properties of the real PMS of Type IV for AR(2) and ARMA(1,1), in terms of the model parameters, surface pointedly in this study, adding up to one already enunciated by [8] for the AR(1) model — $\text{PMS}_{\text{IV}}(\delta, \phi_1, \phi_2)$ (resp. $\text{PMS}_{\text{IV}}(\delta, \phi, \alpha)$) appears to increase with both ϕ_1 and ϕ_2 (resp. increase with ϕ and decrease with α), when simultaneous residual schemes are used to control the mean and variance of AR(2) (resp. ARMA(1,1)) output. This paper also reaffirms that simultaneous EWMA residuals schemes should be preferred to the Shewhart-type if we plan to anticipate a few dramatic reductions of the PMS of types III and IV.

Misleading signals deserve further investigation while using other simultaneous schemes for μ and σ^2 suchlike the ones pertinently proposed by [5], simultaneous EWMA schemes with the following characteristics: their constituent charts for σ^2 are able to detect both upward and downward shifts in the process variance; the maximum of ARL for fixed $\mu = \mu_0$ and for varying σ^2 is attained at $\sigma^2 = \sigma_0^2$. Future research can also be done in the following direction: assess the impact on MS of falsely assuming a simpler model, e.g., an AR(1) model, when the output is better described by a slightly more complex process, e.g., an AR(2) process or an ARMA(1,1) model.

Since MS can be rather frequent and the general assumption of independence can have a meaningful effect in the ability of a simultaneous scheme for the process mean and variance to identify which one of these two parameters has changed, it is convenient to implement additional procedures for use as diagnostic aids to determine which parameters changes, as recommended by [20]. Although investigation on these diagnostic procedures is beyond the scope of this paper, this issue will be certainly considered in future work and we shall take into account that [18] suggest the use of the pattern of the points beyond the control limits of the constituent charts in the identification of the parameter that has effectively changed (a plausible justification for this diagnostic aid stems from the fact that changes in μ and σ^2 have different impacts in those patterns).

Finally, let us remind the reader that the phenomenon of MS can also arise in other settings, such as multivariate control schemes for the mean vector and the covariance matrix of i.i.d. output as investigated by [17] and [16].

ACKNOWLEDGMENTS

This work received financial support from Portuguese National Funds through *Fundação para a Ciência e a Tecnologia* (FCT) within the scope of the projects PEst-OE/MAT/UI0822/2011 and PTDC/MAT-STA/3169/2012. The second author was supported by grant SFRH/BD/35739/2007 of FCT and was partially supported by *Centro de Matemática e Aplicações* (CEMAT) during the preparation of this paper. The first and second authors would like to thank Professors Sven Knoth and Vasyl Golosnoy, for the pertinent suggestions and questions during the "Statistische Woche 2012, Wien".

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