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ROBUST METHODS IN ACCEPTANCE SAMPLING

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Abstract:

- In the quality control of a production process (of goods or services), from a statistical point of view, the focus is either on the process itself with application of Statistical Process Control or on its frontiers, with application of Acceptance Sampling (AS) and Experimental Design. AS is used to inspect either the process output (final product) or the process input (raw material). The purpose of the design of a sampling plan is to determine a course of action that, if applied to a series of lots of a given quality, and based on sampling information, leads to a specified risk of accepting/rejecting them. Thus AS yields quality assurance. The classic AS by variables is based on the hypothesis that the observed quality characteristics follow the Gaussian distribution (treated in classical standards). This is sometimes an abusive assumption that leads to wrong decisions. AS for non-Gaussian variables, mainly for variables with asymmetric and/or heavy tailed distributions, is a relevant topic. When we have a known non-Gaussian distribution we can build specific AS plans associated with that distribution. Alternatively, we can use the Gaussian classical plans with robust estimators of location and scale — for example, the total median and the sample median as location estimates, and the full range, the sample range and the interquartile range, as scale estimates. In this work we will address the problem of determining AS plans by variables for Extreme Value distributions (Weibull and Fréchet) with known shape parameter. Classical plans, specific plans and plans using the robust estimates for location are determined and compared.

Key-Words:

- *quality control; acceptance sampling; acceptance sampling by variables; robust methods.*

1. INTRODUCTION

Acceptance Sampling (AS) is used to inspect either the output process — final product — or the input — initial product. On a lot-by-lot basis, a random sample is taken from the lot and based on the information given by the sample a decision is taken: to accept or to reject the lot. The purpose of AS is to determine a course of action, not to estimate lot quality. It prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. An AS plan indicates the rules for accepting or rejecting a lot that is being inspected. Acceptance sampling is a compromise between no inspection and 100% inspection. It is likely to be used under the following conditions:

- When 100% inspection is tiring the percentage of nonconforming items passed may be higher than under a scientifically designed sampling plan.
- When the cost of inspection is high and the loss arising from the passing of a nonconforming unit is not great. It is possible in some cases that no inspection at all will be the cheapest plan.
- When inspection is destructive. In this case sampling must be employed.

There are two approaches to AS in the literature. The first approach is AS by attributes, in which the product is specified as conforming or nonconforming (defective) based on a certain criteria and the number of nonconforming units is counted. The other approach is AS by variables, if the item inspection leads to a continuous measurement. In comparison to sampling plans by attributes, sampling plans by variables have the advantage of usually resulting in considerable savings in sample size for comparable assurance. The main disadvantage of the classical case of the acceptance sampling by variables is that it is based on the hypothesis that the observed quality characteristic follows a Gaussian distribution. References to this section are ([4]), ([12]), ([9]), ([13]).

In Acceptance Sampling there are two kinds of decisions based on the sample, to accept or to reject the lot, and two kinds of errors associated:

- Type I error: consists of incorrectly rejecting a lot that is really acceptable. The probability of making a type I error is α, also called *producer's risk*.
- Type II error: consists of incorrectly accepting a lot that is really unacceptable. The probability of making a type II error is β, also called *consumer's risk*.

The producer wishes the acceptance of "good" lots with high probability $(1-\alpha)$ and the consumer wishes the acceptance of "bad" lots with small probability (β).

In the determination of an AS plan the aim is to calculate the sample size, n, to be taken from the lot and the acceptability constant, k, that satisfy the conditions referred to as the *producer's risk* and the *consumer's risk*. There are two quality values that we need to define ([14]):

- AQL — *Acceptable Quality Level* — the worst quality level that is still considered acceptable. The AQL is a percent defective that is the base line requirement for the quality of the producer's product.
- *LTPD* — *Lot Tolerance Percent Defective* — the poorest quality in an individual lot that should be accepted, the level of quality where it is desirable to reject most lots. The *LTPD* is a designated high defect level that would be unacceptable to the consumer.

To prevent "good" lots from being rejected and "bad" lots from being accepted, we calculate the values of n and k by solving the system

$$\begin{cases} P_{\mathrm{ac}}(\omega = AQL) \;=\; 1-\alpha\,, \\ P_{\mathrm{ac}}(\omega = LTPD) \;=\; \beta\,, \end{cases} \tag{1.1}$$

where $P_{\mathrm{ac}}(\omega) = \mathrm{P}(\text{accept the lot}\,|\,\omega)$ designates the acceptance probability (function of n and k) and ω the non conforming proportion. If we let ω vary in $[0,1]$, we can establish the operating characteristic curve, *OC-curve*, $P_{\mathrm{ac}}(\omega)$. This curve shows the lot acceptance probability in accordance with its quality, given by the nonconforming proportion. This is the most used way of determining an AS plan: to specify 2 desired points on the *OC-curve* and solve for the (n,k) that uniquely determines the *OC-curve* going through these points $(AQL, 1-\alpha)$ and $(LTPD, \beta)$ ([8]). Alternatively the above system can be solved for k and *LTPD*, as will be used later for comparison purposes.

In AS, sampling plans can be built up with a single specification limit (the upper or lower) or with two specification limits (the upper and the lower). This latter situation is theoretically more complex since the two previous procedures have to be added into one. For more details see ([3]).

Let X denote the random variable that represents the quality characteristic inspected. For simplicity, in the next sections we will assume that there is a single specification limit, the upper limit U, so the nonconforming proportion is given by $\omega = P(X \geq U)$. In section 2 we will review the classical case where X is assumed to follow a Gaussian distribution. In section 3 we will derive AS plans when X follows an Extreme Value distribution (Weibull and Fréchet). As a particular case of Weibull distribution we obtain the results for the exponential distribution studied in ([2]) and ([11]). In the section 4 robust estimators for location are presented. In section 5, classical plans, specific plans and plans using the robust estimates for location are compared by means of the *OC-curve*. The main conclusions are driven in section 6.

2. ACCEPTANCE SAMPLING FOR GAUSSIAN VARIABLES

The acceptance sampling by variables in the Gaussian case was solved in theory and for application in American Standard, MIL-STD 414 (updated several times in details). The most recent international version is ([1]).

Consider that the quality characteristic of interest, X, follows a Gaussian distribution, with mean μ and standard deviation σ, $X \frown N(\mu, \sigma)$ and that a sample of size n is taken from the lot for AS purposes. The nonconforming proportion is given by $\omega = P(X \geq U) = 1 - \Phi\big(\frac{U-\mu}{\sigma}\big)$. The lot is accepted if the estimated nonconforming proportion based on the sample is "small" or an associated quality index Q is "big". The definition of Q depends on the standard deviation of X being known or unknown, as follows.

2.1. σ known

If σ is known the quality index Q is defined as $Q = \frac{U-\overline{X}}{\sigma}$ and the criterion of acceptance is $Q = \frac{U-\overline{X}}{\sigma} \geq k$. The values of n and k are the solution of the system

$$\begin{cases} P\big(Q \geq k \,|\, \omega = AQL\big) \;=\; 1-\alpha\,, \\ P\big(Q \geq k \,|\, \omega = LTPD\big) \;=\; \beta\,, \end{cases}$$

and are given by

$$\begin{cases} k \;=\; \dfrac{z_{1-\alpha}\, z_{LTPD} - z_\beta\, z_{AQL}}{z_\beta - z_{1-\alpha}}\,, \\ n \;=\; \left(\dfrac{z_{1-\alpha} - z_\beta}{z_{LTPD} - z_{AQL}}\right)^2, \end{cases}$$

where z_p denotes the p-probability quantile of the standard Gaussian distribution. For details see ([4]) and ([12]).

2.2. σ unknown

If σ is unknown the criterion of acceptance is $Q = \frac{U-\overline{X}}{S} \geq k$, with $S^2 = \sum_{i=1}^{n} \frac{(X_i-\overline{X})^2}{n-1}$ the unbiased estimator of σ^2. The values of n and k result from the resolution of the system

$$\begin{cases} P_{\rm ac}\big(Q \geq k \,|\, \omega = AQL\big) \;=\; 1-\alpha\,, \\ P_{\rm ac}\big(Q \geq k \,|\, \omega = LTPD\big) = \beta\,, \end{cases}$$

or from the equivalent

$$\begin{cases} F_{t,\nu=n-1,\delta=\sqrt{n}\, z_{AQL}}\big(-k\sqrt{n}\big) \;=\; 1-\alpha\,, \\ F_{t,\nu=n-1,\delta=\sqrt{n}\, z_{LTPD}}\big(-k\sqrt{n}\big) \;=\; \beta\,, \end{cases}$$

where $F_{t,\nu,\delta}(\cdot)$ represents the distribution function of the non-central t variable with ν degrees of freedom and non-centrality parameter δ ([4]) and ([12]).

3. ACCEPTANCE SAMPLING FOR NON-GAUSSIAN VARIABLES

At this point, we will take a closer look at two specific distributions that are widely used in Statistical Quality Control, namely the Weibull and Fréchet distributions.

The procedure to be used for non-Gaussian variables is analogous to that used for the Gaussian case. We start by defining the quality index for each case and compare its observed value with the constant k of acceptance, considering the situations of known and unknown parameters. To define the AS plan for each case, we must solve system (1.1).

Let us consider the Weibull distribution, Weibull(θ, δ), with probability density function (*pdf*) $f_X(x) = \frac{\theta}{\delta}\left(\frac{x}{\delta}\right)^{\theta-1} e^{-\left(\frac{x}{\delta}\right)^{\theta}}$, $x > 0$, $\delta > 0$, $\theta > 0$, and the Fréchet distribution, Fréchet(θ, δ), with *pdf* $f_X(x) = \frac{\theta}{\delta}\left(\frac{x}{\delta}\right)^{-\theta-1} e^{-\left(\frac{x}{\delta}\right)^{-\theta}}$, $x > 0$, $\delta > 0$, $\theta > 0$, and let $\hat{\theta}$ and $\hat{\delta}$ represent the maximum likelihood estimators of their respective dispersion and shape parameters based on a random sample of size n. Considering that $Y = \frac{2n\,\hat{\delta}^{\theta}}{\delta^{\theta}} \frown \chi^2_{2n}$, in the Weibull case, and that $Y = \frac{2n\,\hat{\delta}^{-\theta}}{\delta^{-\theta}} \frown \chi^2_{2n}$, in the Fréchet case, the results of Table 1 are obtained (for details see ([3])).

Table 1: Non-Conforming proportion, Criterion of acceptance and Acceptance Sampling plans for the Weibull and Fréchet cases.

Distribution		Weibull (θ, δ)	Fréchet (θ, δ)
Non-Conforming Proportion $\omega = P(X > U)$		$e^{-\left(\frac{U}{\delta}\right)^{\theta}}$	$1 - e^{-\left(\frac{U}{\delta}\right)^{-\theta}}$
Criterion of acceptance	θ known	$Q_U = \left(\frac{U}{\widehat{\delta}}\right)^{\theta} \geq k$	$Q_U = \left(\frac{U}{\widehat{\delta}}\right)^{-\theta} \leq k$
	θ unknown	$Q_U = \left(\frac{U}{\widehat{\delta}}\right)^{\widehat{\theta}} \geq k$	$Q_U = \left(\frac{U}{\widehat{\delta}}\right)^{-\widehat{\theta}} \leq k$
AS plans: values of k and n	θ known	$\begin{cases} n = -\dfrac{k\,\chi^2_{2n;\beta}}{2\,ln(LTPD)} \\ k = -\dfrac{2n\,ln(AQL)}{\chi^2_{2n;1-\alpha}} \end{cases}$ *	$\begin{cases} n = -\dfrac{k\,\chi^2_{2n;1-\beta}}{2\,ln(1-LTPD)} \\ k = -\dfrac{2n\,ln(1-AQL)}{\chi^2_{2n;\alpha}} \end{cases}$
	θ unknown	**	**

* Note that, this system is equal to the exponential case, ([2]).

** Since the exact distribution of Q_U is unknown analytically, to determine the values of n and k that satisfy the system (1.1) we have to proceed with simulation methods.

4. ROBUST ESTIMATORS FOR LOCATION

As we referred previously, when we have a non-Gaussian distribution we can build specific AS plans associated with that distributions.

As also mentioned, the classical plans (Gaussian case) assume normality of the data and they use $\overline{X}$ as an estimator of μ. However, when data is Non-Gaussian, $\overline{X}$ may not be the best estimator, mainly when the distribution is asymmetric and/or has heavy tails.

Thus, alternatively, as robust estimators for μ, we suggest the sample median $\big(\widetilde{X}\big)$ and total median $\big(\widetilde{X}_T\big)$, respectively, given by

$$\widetilde{X} = \begin{cases} X_{(m)} & \text{if } n = 2m-1\,, \\ \dfrac{X_m + X_{m+1}}{2} & \text{if } n = 2m\,, \end{cases}$$

$m \geq 1$ and $\widetilde{X}_T = \sum_{i=1}^n a_i X_{(i)}$, such that $a_i = a_{n-i+1}, \forall\, i = 1, ..., n,\ 0 < a_1 < a_2 < ... < a_{\left[\frac{n}{2}\right]},\ \sum_{i=1}^n a_i = 1$.

Considering these estimators for the mean value, the quality index of classical plans is, respectively $Q_N' = \frac{U - \widetilde{X}}{\sigma}$ and $Q_N'' = \frac{U - \widetilde{X}_T}{\sigma}$, and the criterion of acceptance, for each case is $Q_N' = \frac{U - \widetilde{X}}{\sigma} \geq k_n'$ and $Q_N'' = \frac{U - \widetilde{X}_T}{\sigma} \geq k_n''$. When we work with Q_N', we use the distribution of $\widetilde{X}$. When we work with Q_N'', we need to use simulation methods, since its distribution is unknown.

For calculating the weight of the tails, we used the index, τ,([10]),

$$\tau(F) = \frac{1}{2}\left(\frac{F^{-1}(0.99) - F^{-1}(0.5)}{F^{-1}(0.75) - F^{-1}(0.5)} + \frac{F^{-1}(0.5) - F^{-1}(0.01)}{F^{-1}(0.5) - F^{-1}(0.25)}\right) \Big/ \left(\frac{z_{0.99} - z_{0.5}}{z_{0.75} - z_{0.5}}\right),$$

where $F^{-1}(p)$ represents the p-quantile of the distribution F and z_p represents de the p-quantile of the standard Gaussian distribution.

To assess the degree of skewness, Fisher's skewness coefficient, c_1, was used, given by $c_1(F) = \frac{\mu_3}{\sigma^3}$, where F represents the distribution of the data, μ_3 represents the third-order central moment of the distribution F and σ represents the standard deviation of the distribution F.

According to ([7]), for asymmetric distributions, the best estimator for the mean value is

- $\overline{X}$, if $c_1 < 0.9$ independently of the value of n or if $n > 16$ independently of the value of c_1;

- $\widetilde{X}_T$, if $0.9 \leq c_1 \leq 3.69$ and $n \leq 16$;
- $\widetilde{X}$, if $c_1 > 3.69$ and $n = 3$ or 4;
- $\widetilde{X}_T$, if $c_1 > 3.69$ and $5 \leq n \leq 16$.

For the tail weight index (τ), the best estimator for the mean value is

- $\overline{X}$, if $\tau < 1.01$ independently of the value of n;
- $\widetilde{X}_T$, if $1.01 \leq \tau \leq 1.8$ and $n \geq 5$;
- $\widetilde{X}$, if $\tau > 1.8$ and $n = 3$ or 4;
- $\widetilde{X}_T$, if $\tau > 1.8$ and $n \geq 5$.

5. SOME RESULTS

Our main questions are: what miscalculations occur if X is Weibull and we use a standard AS plan for Gaussian X instead? What alternatives can we use? Can we use robust estimators for the location in the Gaussian case?

As we said before, the determination of the specific sampling plan is based on the solution of the System (1.1). Usually α, β, AQL and *LTPD* are fixed and the system is solved for n and k. For comparison of the plans it is more convenient to fix n (taken from the standard) and solve the system to calculate k and *LTPD*. The comparison of the results will, essentially, be based on *LTPD* or/and the *OC-curve*.

To exemplify what we propose, we consider distributions with different degrees of asymmetry and tail weight index. So we are going to compare the Gaussian case with the Weibull ($\theta = 7$ and $\theta = 1$) and Fréchet ($\theta = 5$) cases. We will consider $\alpha = 5\%$, $\beta = 10\%$, $AQL = 1\%$ and several values of n, taken from the standard.

5.1. Comparisons of Gaussian and specific plans

If the quality characteristic is a non-Gaussian variable and if we use the values of the standard (apply the classical plans), the producers risk (5%) is miscalculated and misleading. We have, therefore, to carry out the adjustment of the α's for the *OC-curves* which pass in the point $(AQL, 1-\alpha)$, and so we can compare the sampling plans. For more details see ([3]). Table 2 shows the results for the exponential case.

Table 2: Results of α's adjustment, exponential case.

Sample size, n	Adjusted α
10	0.036
15	0.038
20	0.039
30	0.041
35	0.041
50	0.043
75	0.044
100	0.045
150	0.046
200	0.046

Table 3 shows the comparison results of the Gaussian case (given by the standard) versus the exponential case, based on *LTPD* and k.

Table 3: Comparison of *LTPD* and k, between Gaussian case (σ known) and exponential case.

Sample size, n	$AQL = 1\%$					
	Standard (α is not 5%) Gaussian data Gaussian fit		Exponential data Gaussian fit		Exponential data Exponential fit	
	$LTPD_N(\%)$	k_N	$LTPD_{EN}(\%)$	k_{EN}	$LTPD_E(\%)$	k_E
10	8.06	1.81	16.13	1.76	16.13	2.93
15	5.81	1.90	11.45	1.87	11.45	3.16
20	4.73	1.96	9.08	1.93	9.08	3.30
30	3.66	2.03	6.68	2.01	6.68	3.49
35	3.35	2.05	5.99	2.03	5.99	3.56
50	2.79	2.09	4.73	2.08	4.73	3.70
75	2.34	2.14	3.73	2.13	3.73	3.85
100	2.10	2.16	3.20	2.16	3.20	3.94
150	1.84	2.19	2.65	2.19	2.65	4.05
200	1.70	2.21	2.36	2.21	2.36	4.12

Examining the results presented in Tables 2 and 3, it can be seen that if the quality characteristic is an exponential variable and if we use the values of the standard (classic case), the producer's risk (as well as the consumer's) is miscalculated. For example, given $AQL = 1\%$, $n = 10$ and if we want a producer's risk of 5%, standards give the values of k and *LTPD*, respectively, 1.81 and 8.06%.

But in fact, with this k the real risk of the producer is 6.36% (the risk of 5% is illusory and misleading) and the real consumer's non-conforming fraction is 16.13% (instead of 8.06%). Therefore, to ensure a risk of 5% the standard shall be calculated with a risk of 3.6%, yielding the acceptance constant, k, in the last but one column of Table 3.

Tables 4 and 5 show the results of the Weibull case with $\theta = 7$. These results show, once again, that abusively using AS plans for Gaussian variables,

Table 4: Simulation results: estimated α for Gaussian case when α of Weibull ($\theta=7$, $\delta=10$) case is 0.05 and 95% Confidence Interval for α.

Sample size, n	Adjusted α	95% Confidence Interval for α	
		Lower limit	Upper limit
10	0.055	0.042	0.069
15	0.053	0.041	0.068
20	0.053	0.040	0.067
30	0.053	0.041	0.067
35	0.052	0.039	0.066
50	0.052	0.040	0.066
75	0.052	0.040	0.065
100	0.051	0.039	0.065
150	0.051	0.039	0.066
200	0.051	0.038	0.066

Table 5: Comparison of *LTPD* and k, between the Gaussian case and the Weibull ($\theta = 7$, $\delta = 10$) case.

Sample size, n	$AQL = 1\%$					
	Standard (α is not 5%) Gaussian data Gaussian fit		Weibull data Gaussian fit		Weibull data Weibull fit	
	$LTPD_N(\%)$	k_N	$LTPD_{WN}(\%)$	k_{WN}	$LTPD_W(\%)$	k_W
10	8.06	1.81	21.00	1.76	16.13	2.93
15	5.81	1.90	14.00	1.87	11.45	3.16
20	4.73	1.96	12.00	1.93	9.08	3.30
30	3.66	2.03	9.00	2.01	6.68	3.49
35	3.35	2.05	7.50	2.03	5.99	3.56
50	2.79	2.09	6.00	2.08	4.73	3.70
75	2.34	2.14	5.00	2.13	3.73	3.85
100	2.10	2.16	4.00	2.16	3.20	3.94
150	1.84	2.19	3.00	2.19	2.65	4.05
200	1.70	2.21	2.36	2.21	2.36	4.12

when we have an exponential or Weibull variable, implies serious risks for the consumer and/or the producer. Comparing the last column of Tables 3 and 5, we can see that the results for the exponential and Weibull cases are equal, i.e., the plans are θ invariant.

The same kind of precautions has to be taken in the Fréchet distribution for the calculation of the risks α and β, and the constants k and $LTPD$.

5.2. Comparisons of specific and robust AS plans

The plots in Figures 1 and 2 show, for $n = 5$, the operating characteristic curves, *OC-curves*, $P_{\rm ac}(\omega)$ for:

- the Gaussian case with sample mean (–○–);
- the Gaussian case with sample median (Figure 1) and total median (Figure 2) (–□–) with Weibull data;
- Weibull case with $\theta = 7$, $\delta = 10$ (–●–). This distribution has $c_1 = -0.463$ and $\tau = 0.990$.

Observing the graphs of Figures 1 and 2, it appears that the mean sample produces better results than the sample median or the total median. The *OC-curve* of the Gaussian case with sample mean is closer to the specific case and is below of the *OC-curves* of the Gaussian case with sample median and total median. For other values of n, the results are similar. $\overline{X}$ is the best estimator for this type of distribution.

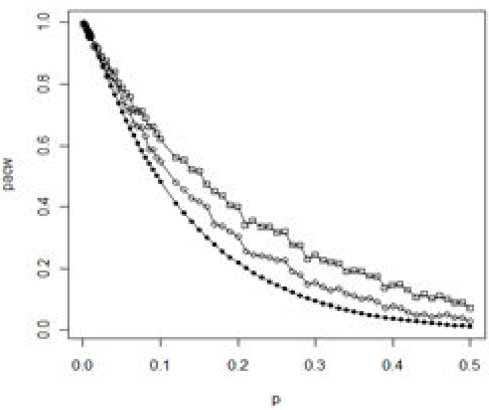

Figure 1: Comparison of OC-curves, $P_{\rm ac}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Weibull (simulated values) and Gaussian case, $n = 5$:
(–□–) Gaussian case with σ known and sample median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Weibull case with θ known.

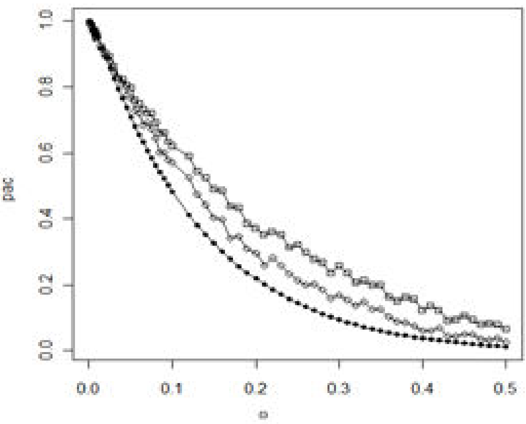

Figure 2: Comparison of OC-curves, $P_{ac}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Weibull (simulated values) and Gaussian case, $n=5$:
(–□–) Gaussian case with σ known and total median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Weibull case with θ known.

The plots in Figures 3 and 4 show, for $n = 5$, the *OC-curves*, $P_{ac}(\omega)$ for:

- the Gaussian case with sample mean (–○–);
- the Gaussian case with sample median (Figure 3) and total median (Figure 4) (–□–) with Weibull data;
- Weibull case with $\theta = 1$, $\delta = 10$ (exponential case) (–●–). This distribution has $c_1 = 6.619$ and $\tau = 2.260$.

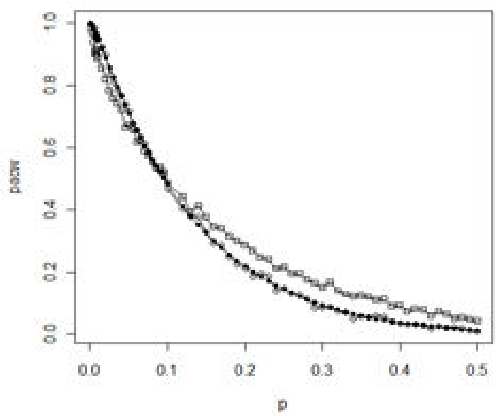

Figure 3: Comparison of OC-curves, $P_{ac}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Weibull $(\theta=1)$ (simulated values) and Gaussian case, $n=5$:
(–□–) Gaussian case with σ known and sample median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Weibull case with θ known.

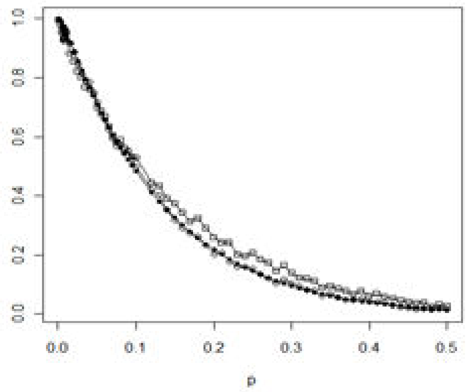

Figure 4: Comparison of OC-curves, $P_{\rm ac}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Weibull ($\theta = 1$) (simulated values) and Gaussian case, $n = 5$:
(–□–) Gaussian case with σ known and total median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Weibull case with θ known.

In this special case (exponential) $\overline{X}$ is the best estimator for this type of distribution, it produces the best results. This is a special case, since it contradicts the results obtained by ([7]). We can see that, after adjusting for the α's, the *OC-curves* of the specific case and the classic case with mean sample are coincident, and there is, therefore no alternative to improve the results. For other values of n, the results are similar.

The plots in Figures 5 and 6 show, for $n = 5$, the *OC-curves*, $P_{\rm ac}(\omega)$ for:

- the Gaussian case with sample mean (–○–);
- the Gaussian case with sample median (Figure 5) and total median (Figure 6) (–□–) with Fréchet data;
- Fréchet case with $\theta = 5$, $\delta = 10$ (–●–). This distribution has $c_1 = 3.535$ and $\tau = 1.357$.

In this case $\widetilde{X}_T$ is the best estimator for this type of distribution, i.e., we get the best results relatively to $\widetilde{X}$ and $\overline{X}$. For other values of n, the results are similar.

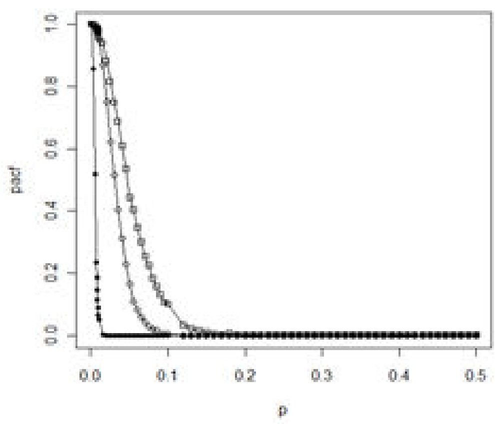

Figure 5: Comparison of OC-curves, $P_{\text{ac}}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Fréchet ($\theta = 1$) (simulated values) and Gaussian case, $n = 5$:
(–□–) Gaussian case with σ known and sample median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Fréchet case with θ known.

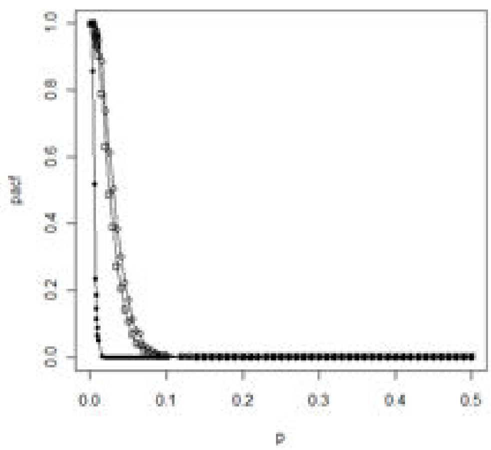

Figure 6: Comparison of OC-curves, $P_{\text{ac}}(p)$, $p \in [0;1]$ — range of non-conforming proportion — between Fréchet ($\theta = 1$) (simulated values) and Gaussian case, $n = 5$:
(–□–) Gaussian case with σ known and total median;
(–○–) Gaussian case with σ known and sample mean;
(–●–) Fréchet case with θ known.

6. CONCLUSIONS

It is important to note that standard sampling plans by variables are not to be used indiscriminately, when the normality assumption may be questioned. Application of an incorrect sampling plan can cause damage to the producer and to the consumer.

If data comes from the Weibull model with $\theta = 1$, i.e., the exponential case, and if we apply the standard k determined for the Gaussian case, the producer's risk (level) of the AS plan will no longer be 5%, but will be lower, what is convenient for the producer. The values of the *LTPD*, important for the consumer, are also miscalculated when using the wrong model. However, after adjusting the α's, the AS plans are equal.

If data comes from the Weibull model with $\theta \neq 1$ and we use the appropriate AS plan (considering this distribution), as expected, we get better results than if we use the standard AS plan (assuming Gaussian case), as the *OC-curve* for the Weibull plan is below the one for the standard plan (Figure 1).

The results of using the statistics Q'_N and Q''_N are (except in the exponential case) in agreement with those obtained by ([7]), ([5]) and ([6]), i.e., the efficiency of the robust estimators for location depends on the asymmetry and the tail weight of the data distribution. When the distribution of the quality characteristic is Weibull, $\theta = 7$, so has a low skewness coefficient and a low tail weight index (Figures 1 and 2), $\overline{X}$ produces better results than the $\widetilde{X}$ and the $\widetilde{X}_T$, as expected. When the distribution of the quality characteristic is Fréchet, $\theta = 5$, so has a high skewness coefficient and a high tail weight index (Figures 5 and 6), $\widetilde{X}_T$ produces better results than the $\overline{X}$ and the $\widetilde{X}$.

So, when faced with the problem of determining AS plans for quality characteristics with non-Gaussian variables but we are able to adequately model the data and estimate its parameters, which usually is not easy, we can use specific AS plans. Alternatively, mainly for variables with asymmetric and/or heavy tailed distributions, robust AS plans are to be considered as a good alternative to the classical plans.

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REFERENCES

[1] ANSI/ASQC Z1.9-2011 (2011). *Sampling Procedures and Tables for Inspection by Variables for Percent Nonconforming*, ASQ, Milwaukee, WI (USA).

[2] Carolino, E.; Casquilho, M. and Barão, M. (2007). Amostragem de aceitação para uma variável assimétrica: a Exponencial, *Actas do XIV Congresso Anual da Sociedade Portuguesa de Estatística*, 281–292.

[3] Carolino, Elisabete (2012). *Amostragem de Aceitação para Variáveis não Gaussianas*, PhD Dissertation (in portuguese), FCUL, Portugal.

[4] Duncan, A.J. (1986). *Quality Control and Industrial Statistics*, 5th edition, IRWIN, USA.

[5] Figueiredo, F. and Gomes, M.I. (2004). The total median in statistical quality control, *Applied stochastic models in business and industry*, **20**, 339–353.

[6] Figueiredo, F. and Gomes, M.I. (2009). Monitoring industrial processes with robust control charts, *Revstat*, **7**, 151–170.

[7] Figueiredo, F.O. (2003). *Controlo Estatístico da Qualidade e Métodos Robustos*, PhD Dissertation (in portuguese), FCUL, Portugal.

[8] Gomes, M.I.; Figueiredo, F. and Barão, M.I. (2010). *Controlo Estatístico da Qualidade*, Segunda Edição revista e aumentada, Edições SPE.

[9] Guenther, William C. (1977). *Sampling Inspection in Statistical Quality Control*, First published, Whitstable Litho Ltd, GB.

[10] Hoaglin, David C.; Mosteller, Frederick and Tuckey, John W. (2000). *Understanding Robust and Explanatory Data Analysis*, John Wiley and Sons, Inc., New York.

[11] Levinson, W. (1997). Watch out for non-normal distributions of impurities, *Chemical Engineering Progress*, 70–76.

[12] Montgomery, D.C. (2004). *Introduction to Statistical Quality Control*, 5th edition, John Wiley and Sons, Inc., New York, USA.

[13] Schilling, E.G. and Neubauer, D.V. (2009). *Acceptance sampling in Quality Control*, 2nd ed., Chapman and Hall / CRC, New York, USA.

[14] Wetherill, G.B. and Brown, D.W. (1991). *Statistical Process Control*, Chapman and Hall, London, UK.