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PREDICTIVE ESTIMATION OF POPULATION MEAN IN RANKED SET SAMPLING

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Abstract:

- The article presents predictive estimation of population mean of the study variable in Ranked Set Sampling (RSS). It is shown that the predictive estimators in RSS using mean per unit estimator, ratio estimator and regression estimator as predictor for non-sampled values are equivalent to the corresponding classical estimators in RSS. On the other hand, when product estimator is used as predictor, the resulting estimator differs from the classical product estimator under RSS. Expressions for the Bias and the Mean Squared Error (MSE) of the proposed estimators are obtained up to first order of approximation. A simulation study is conducted to observe the performance of estimators under predictive approach.

Key-Words:

- *efficiency; product estimator; ratio estimator; regression estimator; RSS.*

1. INTRODUCTION

It is very common to construct estimators for population parameters of a study variable using the information contained only in a sample of the study variable. However, in many situations, statisticians are interested in using some auxiliary information from the population itself which helps in finding more efficient estimators. In literature, a lot of work has been done on how to use the auxiliary information (see for example, Agrwal and Roy (1999), Upadhyaya and Singh (1999), Singh (2003), Singh and Tailor (2003), Kadilar and Cingi (2004, 2006), Yan and Tian (2010), and Singh *et al.* (2014)). In many situations, we may be interested in estimating the average value of the variable being measured for non-sampled units on the basis of sample data at hand. This approach is called predictive method of estimation. This approach is based on superpopulation models, and hence it is also called model-based approach. The approach assumes that the population under consideration is a realization of random variables following a superpopulation model. Under this model the prior information about the population parameters such as the mean, the variance, and other parameters is utilized to predict the non-sampled values of the study variable.

Basu (1971) constructed predictive estimators for population mean using mean per unit estimator, regression estimator, and ratio estimator as predictors for the mean of unobserved units in the population. Srivastava (1983) compared the estimator obtained by using the product estimator as a predictor for mean of unobserved units in the population with the customary product estimator. Recently, Yadav and Mishra (2015) have established predictive estimators using product estimator as predictor for the mean of unobserved units of the population.

Basic statistical principles play a vital role in making inference about the population of interest. If these principles are violated, even optimal statistical procedures will not allow us to make legitimate statistical inferences about the parameters of interest. Ranked Set Sampling (RSS) technique is a good alternative for Simple Random Sampling (SRS) for obtaining experimental data that are truly representative of the population under investigation. This is true across all of the sciences including agricultural, biological, environmental, engineering, physical, medical, and social sciences. This is because in RSS measurements are likely more regularly spaced than measurements in SRS. The RSS procedure creates stratification of the entire population at the sampling stage, i.e. we are randomly selecting samples from the subpopulations of small, medium and large units without constructing the subpopulation strata in advance. Ranked set sampling method, proposed originally by McIntyre (1952) to estimate mean pasture yields, has recently been modified by many authors to estimate the population parameters. Dell and Clutter (1972) showed that the mean estimator is an unbiased estimator of the population mean under RSS for both perfect as well as imperfect ranking. Muttlak (1997) suggested median ranked set sampling (MRSS) for estimation of finite population mean. Al-Saleh and Al-Omari (2002) used multistage ranked set sampling (MSRSS) to increase the efficiency of the estimator of the population mean for certain value of the sample size. Jemain and Al-Omari (2006) suggested double quartile ranked set sampling (DQRSS) for estimating the population mean. Many other authors have worked on estimation of parameters in RSS (see Al-Omari and Jaber (2008), Bouza (2002), Al-Nasser (2007), Ohyama *et al.* (1999), and Samawi and Muttlak (1996) among others).

In this study, we propose a predictive estimator, using ratio, product and regression estimators as predictors for non-sampled observations under ranked set sampling scheme. In Section 2, we review the predictive estimators introduced by Basu (1971). Section 3 consists of the proposed estimators and their properties. An efficiency comparison is carried out through simulations in Section 4. Some concluding remarks are given in Section 5.

2. PREDICTIVE ESTIMATORS IN SIMPLE RANDOM SAMPLING

Let $U = \{U_1, U_2, ..., U_N\}$ be a population of size N. Let (y_i, x_i) be the values of the study variable y and the auxiliary variable x on the i-th $(0 \leq i \leq N)$ unit of U.

Let S be the set of all possible samples from U using simple random sampling with replacement (SRSWR). For any given $s \in S$, let $\vartheta(s)$ be the number of distinct units in s and let $\bar{s}$ denote the set of all those units of U which are not in s. Basu (1971) presented population mean as follows:

$$\bar{Y} = \frac{\vartheta(s)}{N}\bar{Y}_s + \frac{N-\vartheta(s)}{N}\bar{Y}_{\bar{s}}, \tag{2.1}$$

where $\bar{Y}_s = \frac{1}{\vartheta(s)}\sum_{i\in s} y_i$ and $\bar{Y}_{\bar{s}} = \frac{1}{N-\vartheta(s)}\sum_{i\in \bar{s}} y_i$. Under simple random sampling with size $\vartheta(s) = n$, the predictor for overall population mean is given by

$$\bar{Y} = \frac{n}{N}\bar{Y}_s + \frac{N-n}{N}\bar{Y}_{\bar{s}}, \tag{2.2}$$

where $\bar{Y}_s = \frac{1}{n}\sum_{i\in s} y_i$ and $\bar{Y}_{\bar{s}} = \frac{1}{N-n}\sum_{i\in \bar{s}} y_i$. An appropriate estimator of the population mean is then given by

$$t = \frac{n}{N}\bar{y}_s + \frac{N-n}{N}T, \tag{2.3}$$

where T is the predictor of $\bar{Y}_{\bar{s}}$. Basu (1971) used the mean per unit estimator $\bar{y} = \frac{1}{n}\sum_{i\in s} y_i$, ratio estimator $\bar{y}_r = \frac{\bar{y}_s}{\bar{x}_s}\bar{X}_{\bar{s}}$, product estimator $\bar{y}_p = \frac{\bar{y}_s}{\bar{X}_{\bar{s}}}\bar{x}_s$ and regression estimator $\bar{y}_{lr} = \bar{y}_s + \beta(\bar{X}_{\bar{s}} - \bar{x}_s)$ as predictors. Here, $\bar{X}_{\bar{s}} = \frac{1}{N-n}\sum_{i\in \bar{s}} x_i =$

$\frac{N\bar{X}-n\bar{x}_s}{N-n}$ and $\beta = {S_{yx}}\big/{S_x^2}$, where β is regression coefficient of Y on X, and $\bar{X}$ is the population mean of the auxiliary variable based on N units both are assumed to be known in advance. Also, let $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ and $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})$.

It has been shown by Basu (1971) that while using simple mean per unit estimator, ratio estimator and regression estimator as T, the predictive estimator t becomes the corresponding classical simple mean estimator $\bar{y}$, ratio estimator $\bar{y}_r$ and regression estimator $\bar{y}_{lr}$respectively. However, when product estimator is used, then t becomes

$$t_p = \bar{y}_s \frac{n\bar{X} + (N-2n)\bar{x}_s}{N\bar{X} - n\bar{x}_s}. \tag{2.4}$$

It can be easily noticed that t_p is quite different from the usual product estimator.

The Bias and Mean Squared Error (MSE) of t with ratio and product estimators as predictor are given below up to 1^{st} order of approximation:

$$Bias(t_r) \cong \bar{Y}\frac{1}{n}\left(C_x^2 - \rho C_y C_x\right), \tag{2.5}$$

$$Bias(t_p) \cong \bar{Y}\frac{1}{n}\left(\theta C_x^2 + \rho C_y C_x\right) \tag{2.6}$$

and

$$MSE(t_r) \cong \bar{Y}^2\frac{1}{n}\left(C_y^2 + C_x^2 - 2\rho C_y C_x\right), \tag{2.7}$$

$$MSE(t_p) \cong \bar{Y}^2\frac{1}{n}\left(C_y^2 + C_x^2 + 2\rho C_y C_x\right), \tag{2.8}$$

where $C_y = \frac{S_y}{\bar{Y}}$, $C_x = \frac{S_x}{\bar{X}}$, $\rho = \frac{S_{yx}}{S_y S_y}$, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ and $\theta = \frac{n}{N-n}$. Also the bias and *MSE* of t_p are given by

$$Bias(\bar{y}_p) \cong \bar{Y}\frac{1}{n}\left(\rho C_y C_x\right) \tag{2.9}$$

and

$$MSE(\bar{y}_p) \cong \bar{Y}^2\frac{1}{n}\left(C_y^2 + C_x^2 + 2\rho C_y C_x\right). \tag{2.10}$$

From Equations (2.8) and (2.10), it is clear that $\bar{y}_p$ and t_p have same *MSE* when first order of approximation is used although they are different estimators. The variance of t_{lr} is given by

$$Var(t_{lr}) = \frac{1}{n}S_y^2\left(1 - \rho^2\right). \tag{2.11}$$

3. PREDICTIVE ESTIMATOR IN RANKED SET SAMPLING

To obtain a Ranked Set Sample from a superpopulation consisting of N units, an initial sample of m units is selected and ranked according to the attribute of interest. A variety of mechanisms are used for ranking purpose, i.e. visual inspection of units, expert opinion, or through the use of some concomitant variables. If ranking is performed on the auxiliary variable X, the unit that is judged to be the smallest ranked unit from the selected sample is called the first judgment order statistic and is denoted by $Y_{[1]}$. On the other hand, when ranking is performed on the study variable Y itself, the smallest ranked unit (called smallest order statistic) is selected from the sample and denoted by $Y_{(1)}$. Then a second sample of size m (independent of the first sample) is selected from the population and is ranked in the same manner as the first. From the second sample, we select the unit ranked as the second smallest in the sample (i.e. the second judgment order statistic) and is denoted by $Y_{[2]}$ or $Y(2)$ according to the above mentioned definitions. This process continues till inclusion of the largest ranked unit from the m-th sample selected for judgment. This entire process results into m observations and is called a cycle. We complete r cycles to obtain a ranked set sample of size $n = rm$ units.

Let Ω be the all possible samples of size $n = rm$ can be taken from a superpopulation U using a ranked set sampling scheme. Suppose that ω be a single set Ω having size $n = rm$. Let $\bar{\omega}$ denote the set of all those units of U which are not in ω. Let $y_{i[i]}$ and $x_{i(i)}$ be the values of the study variable Y and the auxiliary variable X for i-th unit taken from the i-th judgment ranked sample for actual quantification, where $i = 1, 2, ..., m$. It is assumed that ranking is performed with respect to the auxiliary variable X.

For a ranked set sample of size $n = rm$ (for simplicity, we use $r = 1$), we obtain the following estimators

$$t_{rss[j]} = \frac{m}{N}\bar{y}_{rss} + \frac{N-m}{N}T_{[j]}, (j = 1, 2, 3, 4), \tag{3.1}$$

where $\bar{y}_{rss} = \frac{1}{m}\sum_{i\in\omega} y_{i[i]}$ and $T_{[j]}$ is the predictor for mean of non-sampled observations ($\bar{Y}_{\bar{\omega}}$) which is defined by $T_{[1]} = \bar{y}_{rss}$, $T_{[2]} = \bar{y}_{rss(r)}$, $T_{[3]} = \bar{y}_{rss(lr)}$ and $T_{[4]} = \bar{y}_{rss(p)}$, where $\bar{y}_{rss[r]} = \bar{y}_{rss}\frac{\bar{X}_{\varpi}}{\bar{x}_{rss}}$, $\bar{y}_{rss[lr]} = \bar{y}_{rss} + \beta\left(\bar{X}_{\varpi} - \bar{x}_{rss}\right)$ and $\bar{y}_{rss[p]} = \bar{y}_{rss}\frac{\bar{x}_{rss}}{\bar{X}_{\varpi}}$. Here, $\bar{X}_{\varpi} = \frac{1}{N-m}\sum_{i\in\varpi} x_{i(i)} = \frac{N\bar{X}-m\bar{x}_{rss}}{N-m}$, and $\bar{x}_{rss} = \frac{1}{m}\sum_{i\in\omega} x_{i(i)}$. Inserting $T_{[j]}$ for $(j = 1, 2, 3, 4)$ in Equation (3.1), we have

$$t_{rss[1]} = \bar{y}_{rss}, \tag{3.2}$$

$$t_{rss[2]} = \bar{y}_{rss}\frac{\bar{X}}{\bar{x}_{rss}}, \tag{3.3}$$

$$t_{rss[3]} = \bar{y}_{rss} + \beta(\bar{x}_{rss} - \bar{X}), \tag{3.4}$$

and

$$t_{rss[4]} = \bar{y}_{rss} \frac{m\bar{X} - (N-2m)\bar{x}_{rss}}{N\bar{X} - m\bar{x}_{rss}}. \tag{3.5}$$

Equations (3.2), (3.3) and (3.4) show that $t_{rss[1]}$, $t_{rss[2]}$ and $t_{rss[3]}$ are equivalent to $\bar{y}_{rss}$, $\bar{y}_{rss[r]}$ and $\bar{y}_{rss[lr]}$ respectively. On the other hand $t_{rss[4]}$ differs from $\bar{y}_{rss[p]}$ (usual product estimator under RSS).

To obtain the Bias and the MSE of proposed predictive estimators, we consider the following error terms

$$\in_0 = \frac{\bar{y}_{rss}}{\bar{Y}} - 1 \quad \text{and} \quad \in_1 = \frac{\bar{x}_{rss}}{\bar{X}} - 1$$

such that $E(\in_0) = E(\in_1) = 0$ and

$$E(\in_0^2) = \bar{Y}^{-2}\left(\frac{S_y^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\delta_{y[i]}^2\right),$$
$$E(\in_1^2) = \bar{X}^{-2}\left(\frac{S_x^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\delta_{x(i)}^2\right)$$

and

$$E(\in_0\in_1) = \bar{Y}^{-1}\bar{X}^{-1}\left(\frac{S_{yx}}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\delta_{y[i]}\delta_{x(i)}\right),$$

where $\delta_{y[i]} = \bar{Y}_{[i]} - \bar{Y}$ and $\delta_{x(i)} = \bar{X}_{(i)} - \bar{X}$ for $i = 1, 2, ..., m$. Here, $\bar{Y}_{[i]}$ and $\bar{X}_{(i)}$ are population means of the study variable and the auxiliary variable respectively for i-th order statistic. It is easy to show that $t_{rss[1]}$ is an unbiased estimator of the population mean $\bar{Y}$ with

$$Var(t_{rss[1]}) = \frac{S_y^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\delta_{y[i]}^2. \tag{3.6}$$

It is clear that $Var(t_{rss[1]}) \leq \frac{S_y^2}{m}$. This indicates that $t_{rss[1]}$ is more efficient than $\bar{y}_s$(sample mean under SRSWR). Similarly, the bias and the MSE of $t_{rss[2]}$, up to first order of approximation, are given by

$$Bias(\bar{y}_{rss[2]}) \cong \frac{\bar{Y}}{m}\left[\left(C_x^2 - \rho C_y C_x\right) - \frac{1}{m}\left(\sum_{i=1}^{m}W_{x(i)}^2 - \sum_{i=1}^{m}W_{y[i]}W_{x(i)}\right)\right] \tag{3.7}$$

and

$$MSE(t_{rss[2]}) \cong \frac{1}{m}\left(S_y^2 + R^2S_x^2 - 2R\rho S_y S_x\right) - \frac{1}{m^2}\sum_{i=1}^{m}\kappa_{[i]}^2, \tag{3.8}$$

where $\kappa_{[i]} = W_{y[i]} - RW_{x(i)}$, $W_{y[i]} = \frac{\delta_{y[i]}}{\bar{Y}}$, $W_{x(i)} = \frac{\delta_{x(i)}}{\bar{X}}$ and $R = \frac{\bar{Y}}{\bar{X}}$. From Equations (2.7) and (3.8), it is obvious that $MSE(t_{rss[2]}) \leq MSE(t_r)$, i.e. $t_{rss[2]}$ is more efficient than the predictive type ratio estimator under SRSWR. Further, we can show that $t_{rss[3]}$ is an unbiased estimator of $\bar{Y}$ with variance

$$Var(t_{rss[3]}) = \frac{S_y^2}{m}\left(1-\rho^2\right) - \frac{1}{m^2}\sum_{i=1}^{m} A_{[i]}^2, \tag{3.9}$$

where $A_{[i]} = W_{y[i]} - \beta W_{x(i)}$, $\forall\, i = 1, 2..., m$. Equation (3.9) shows the superiority of the predictive type regression estimator as compared to its counterpart in SRSWR.

Finally, to compute the Bias and the MSE of $t_{rss[4]}$, note that

$$\begin{aligned} t_{rss[4]} &= \bar{Y}(1+\in_0)\frac{m\bar{X} + (N-2m)\bar{X}(1+\in_1)}{N\bar{X} - m\bar{X}(1+\in_1)}, \\ &= \bar{Y}(1+\in_0)\left(1 + \frac{(N-2m)\in_1}{N-m}\right)\left(1 + \frac{m\in_1}{N-m}\right)^{-1}. \end{aligned}$$

Assuming $\left|\frac{m}{N-m}\right| < 1$, and expanding up to first order of approximation using binomial expansion, we have

$$t_{rss[4]} - \bar{Y} \cong \bar{Y}\left(\in_0 + \in_1 + \in_0\in_1 + \phi \in_1^2\right) \tag{3.10}$$

where $\phi = \frac{m}{N-m}$. Taking expectation of Equation (3.10), we get

$$Bias(t_{rss[i]}) \cong \frac{\bar{Y}}{m}\left[C_{yx} + \phi C_x^2 - \frac{1}{m}\sum_{i=1}^{m}\left(\delta_{yx[i]} + \phi\delta_{x(i)}^2\right)\right]. \tag{3.11}$$

MSE of $t_{rss[4]}$ can be obtained by squaring and taking expectation in Equation (3.10). This gives

$$MSE(t_{rss[4]}) \cong \frac{1}{m}\left(S_y^2 + R^2S_x^2 + 2R\rho S_y S_x\right) - \frac{1}{m^2}\sum_{i=1}^{m} B_{[i]}^2, \tag{3.12}$$

where $B_{[i]} = W_{y[i]} + RW_{x(i)}$ for $i = 1, 2, ..., m$.

From Equations (2.6), (2.8), (3.11) and (3.12) it can be noticed that the expression for bias of $t_{rss[4]}$ is different from that of usual product estimator although they have the same MSE for first order of approximation.

4. SIMULATION STUDY

To compare the efficiencies of the proposed estimators, we conduct a simulation study as follows:

1. Generate a hypothetical population on two variables X and Y, where X is generated using three different distributions with some specific values of parameters as described in first row of Table 1.

2. Then Y is generated as $Y = \rho \times X + e$, where e is generated using a standard normal distribution and ρ is the correlation coefficient between X and Y which is fixed at 0.5, 0.7 and 0.9.

3. Take an RSS and a SRSWR, each having size $n = rm$, and compute the proposed estimators and corresponding estimators in SRSWR, where $r = 5, 10$ and $m = 2, 4, 6$.

4. Repeat Step 2, 10,000 times. Then compute the mean squared error of each estimator to obtain relative efficiency of the proposed estimators.

Table 1 provides relative efficiency of proposed predictive estimators in RSS with respect to simple mean estimator in SRS, i.e.

$$RE[j] = \frac{Var(\bar{y}_s)}{MSE(t_{rss[j]})} \quad \text{for} \quad j = 1, 2, 3, 4.$$

Table 1 shows that the relative efficiencies of the RSS increases with the increase of the correlation between the auxiliary variable and the study variable. RE also increases with the increase of the set size m. Predictive estimator using ratio estimator and regression estimator as predictors are almost equally efficient for all the case that considered in this study. However, the product estimator gives worse performance as the correlation between the study variable and the auxiliary variable increases. But this because product estimator is not preferable for prediction in ranked set sampling, when ranking is performed based on an auxiliary variable that has positive correlation with the variable of interest. Efficiencies of the proposed estimators are significantly higher when uniform distribution is used to generate data in the interval [0, 10]. Efficiency is at its peak for uniform distribution with high positive correlation between the study variable and the auxiliary variable.

Table 1: Efficiency Comparison.

	r	m	Normal(5,1)				Exponential(1)				Uniform(0,10)			
			$RE(1)$	$RE(2)$	$RE(3)$	$RE(4)$	$RE(1)$	$RE(2)$	$RE(3)$	$RE(4)$	$RE(1)$	$RE(2)$	$RE(3)$	$RE(4)$
$\rho = 0.5$	5	2	1.1035	1.315	1.3171	0.7292	1.0758	1.0675	1.297	0.6952	1.3359	3.366	3.5067	0.4542
		4	1.1208	1.2557	1.2581	0.8375	1.1616	1.1833	1.3128	0.8434	1.7646	3.5969	3.6251	0.6835
		6	1.223	1.3371	1.3383	0.97	1.1961	1.1827	1.3148	0.9336	2.0576	3.5757	3.5902	0.8916
	10	2	1.0587	1.2869	1.2897	0.691	1.0842	1.0733	1.3092	0.6919	1.3304	3.485	3.6089	0.4504
		4	1.1147	1.2583	1.2598	0.8299	1.1925	1.1805	1.3412	0.8673	1.7856	3.5659	3.6008	0.7013
		6	1.1966	1.3087	1.3097	0.9537	1.1762	1.2036	1.289	0.918	2.048	3.5403	3.5553	0.8895
$\rho = 0.7$	5	2	1.2067	1.9023	1.9053	0.5654	1.1752	1.5186	1.8926	0.5362	1.4461	8.1124	8.4514	0.4061
		4	1.3263	1.8048	1.8084	0.7308	1.3749	1.6819	1.9129	0.7272	2.1256	8.6801	8.7483	0.6425
		6	1.5132	1.9278	1.9295	0.9165	1.4852	1.6825	1.9196	0.8758	2.699	8.5921	8.6269	0.8745
	10	2	1.1361	1.8449	1.849	0.5286	1.1812	1.5155	1.9115	0.5315	1.4348	8.3593	8.6565	0.4023
		4	1.3311	1.8299	1.8321	0.7327	1.4121	1.6694	1.9515	0.7478	2.1744	8.6288	8.7132	0.6629
		6	1.4946	1.8999	1.9013	0.914	1.4611	1.7073	1.8795	0.8616	2.6987	8.5361	8.5724	0.8752
$\rho = 0.9$	5	2	1.3744	4.9954	5.0033	0.4283	1.3408	4.0352*	5.0519*	0.4086	1.5073	33.3303**	34.723**	0.3841
		4	1.7944	4.7412	4.7506	0.6223	1.8154	4.4429*	5.0647*	0.6081	2.3887	35.7477**	36.0284**	0.6244
		6	2.2503	5.0197*	5.0242*	0.8455	2.1969	4.4629*	5.0952*	0.804	3.2381	35.1926**	35.335**	0.8656
	10	2	1.2823	4.8347*	4.8455*	0.4006	1.3338	3.9467*	5.0819*	0.4022	1.4964	34.1889**	35.4045**	0.3803
		4	1.8255	4.8757*	4.8816*	0.6354	1.8519	4.3654*	5.1294*	0.6201	2.4573	35.5361**	35.8836**	0.6447
		6	2.2714	5.0086*	5.0122*	0.8629	2.1789	4.5009*	5.0205*	0.7967	3.2513	35.1079**	35.257**	0.8692

* Stands for higher relative efficiency;
** Stands for highest relative efficiency.

5. CONCLUSION

Assuming a superpopulation model, we developed some predictive type estimators in ranked set sampling as RSS is more efficient method of sample selection for actual measurements. Properties (bias and efficiency) are examined up to first order of approximation. It is observed that the predictive estimators are equivalent to the corresponding classical estimators in RSS when simple mean estimator, ratio estimator and regression estimator are used as predictors for non-sampled values. On the other hand, predictive estimator has different form as compared to the corresponding classical product estimator when product estimator is used as predictor.

This study can be extended by using exponential type estimators and some other efficient estimators as predictor

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