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MODELLING SPATIALLY SAMPLED PROPORTION PROCESSES

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Abstract:

- Many ecological processes are measured as proportions and are spatially sampled. In all these cases the standard procedure has long been the transformation of proportional data with the arcsine square root or logit transformation, without considering the spatial correlation in any way. This paper presents a robust regression model to analyse this kind of data using a beta regression and including a spatially correlated term within the Bayesian framework. As a practical example, we apply the proposed approach to a spatio-temporally sampled fishery discard dataset.

Key-Words:

- *modelling proportions; beta regression; spatial modelling; Bayesian hierarchical modelling.*

1. INTRODUCTION

Many ecological processes are spatially sampled and measured as proportions; one example is, sea-grass coverage in a area. The traditional approach in ecology has been to, first transform proportional data to approximate normality, and then analyse them using Gaussian linear models, such as analysis of variance or linear regression.

A very common transformation is the arcsine square root transformation. This transformation can be useful to stabilise variances and normalise the data but there are several reasons why it should be avoided. Firstly, model parameters cannot be easily interpreted in terms of the original response [Warton and Hui, 2011, Ferrari and Cribari-Neto, 2004]. Secondly, the efficacy of the arcsine transformation in normalising proportional data is heavily dependent on the sample size, and does not perform well at extreme ends of the distribution [Warton and Hui, 2011, Wilson and Hardy, 2002]. Thirdly, measures of proportions typically display asymmetry, and hence inference based on the normality assumption can be misleading [Ferrari and Cribari-Neto, 2004].

An alternative that is becoming more prevalent in ecological analyses is the logistic regression, an analytical method designed to deal with binomial proportional data [Steel *et al.*, 1997, Wilson and Hardy, 2002, Warton and Hui, 2011], i.e. proportions measured as *x out of n*. The logistic regression provides a more biologically and ecologically interpretative analysis and is not sensitive to sample size. Nonetheless, such binomial data is prone to overdispersion, resulting in an incorrect quantification of the uncertainty when applying the proposed binomial generalised linear model (GLM). In these cases, the inclusion of a random intercept term using generalised linear mixed models (GLMMs) may improve the assessment of uncertainty [Wilson and Hardy, 2002].

When data are non-binomial, that is, observations do not follow the *x out of n* pattern, the logistic regression is no longer applicable. As an alternative approach, Warton and Hui [2011] suggested the logit transformation of the data, which overcomes the problems of interpretability and range of the confidence/credible intervals using the arcsine square root transformation. However, any transformation of the data (y_t) implies that regression parameters are only interpretable in terms of the transformed mean of y_t and not the mean of the original data.

The beta distribution is a well known distribution that satisfies the characteristics of proportions, bounded to the [0, 1] interval with asymmetric shapes. It has long been used in a wide range of applications involving proportions and probabilities [Gupta and Nadarajah, 2004]. However, only recently has it been applied to linear regression modelling [Ferrari and Cribari-Neto, 2004, Smithson and Verkuilen, 2006, Liu and Kong, 2015] and time-series analysis [Da-Silva and Migon, 2016], allowing bounded estimates and intervals with model parameters that are directly interpretable in terms of the mean of the response.

Aside from the likelihood function, it is well known that changes in ecological processes in time and space are driven by a set of factors and interactions. Understanding these drivers is very often the ultimate goal among scientists seeking to manage natural resources effectively. However, the immeasurable complexity of ecological spatial processes often means that the spatial variability of the data exceed the variability explained by the explanatory variables. This phenomenon usually results in spatially autocorrelated model residuals that can yield incorrect results and a restricted predictive capacity of the models [Fortin and Dale, 2009, Legendre *et al.*, 2002].

A good solution to improve model fit and prediction is to introduce spatial terms in our models. Spatial terms are based on the principle that close observations have more in common than distant observations [Tobler, 1970]. Consequently, by applying a distance-based function, these terms are capable of improving fine scale predictions and identifying hidden spatial hot and/or cold spots that may be important for management purposes. In addition, from a management perspective it is crucial to address the uncertainty associated with our predictions and estimates. In this respect, the Bayesian hierarchical approach is able to accommodate complex systems and obtain a proper uncertainty assessment by relying on quite straightforward probability rules [Clark, 2005].

The reminder of this article goes as follows. First, we summarise the characteristics of the hierarchical spatial beta regression. Then, we introduce the principles of the Integrated Nested Laplace Approximation (INLA from now on) using the Stochastic Partial Differential Equations (SPDE) approach (`http://www.r-inla.org`) [Rue *et al.*, 2009] as an effective way to deal with spatially sampled proportional data. As an example, we apply this approach to a fishery discards database to identify discard proportion high-density areas in the Western Mediterranean Sea. Finally, we end up with some conclusions.

2. HIERARCHICAL SPATIAL BETA REGRESSION

Traditionally the beta distribution is denoted by two scaling parameters $Be(a, b)$. In order to apply regression it is necessary to reparametrize its density distribution in terms of its mean $\mu = \frac{a}{a+b}$ and a dispersion $\phi = a + b$, so that:

$$\pi(y) = \frac{\Gamma(\phi)}{\gamma(\mu\phi)\gamma(\phi(1-\mu))} \, y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1}\,, \qquad 0 < y < 1\,, \tag{2.1}$$

where Γ is the gamma function, $E(y) = \mu$ and $Var(y) = \frac{\mu(1-\mu)}{1+\phi}$. Note that here, as opposed to the Gaussian distribution, the variance depends on the mean, which translates into maximum variance at the centre of the distribution and minimum at the edges, to support the truncated nature of the beta distribution.

It is also important to note that the probability density (2.1) does not provide a satisfactory description of the data at both ends of the distribution, zero and one. An ad hoc solution may be to add a small error value to the observations to satisfy this criterion [Warton and Hui, 2011]; otherwise zero and one inflated models are required [Liu and Kong, 2015].

Following the $Be(\mu, \phi)$ reparametrisation, a given set of observations $y_1, ..., y_n$, that represent proportions, can be related to a set of covariates and functions using a similar approach to the generalised linear model:

$$\begin{aligned} Logit(\mu_i) &= \eta_i \\ \eta_i &= \alpha + \sum_{j=1}^{n_\beta} \beta_j z_{ji} + \sum_{k=1}^{n_k} f_k(u_{ki}) + v_i \end{aligned} \tag{2.2}$$

where η_i enters the likelihood through a logit link, α is the intercept of the model, β_j are the fixed effects of the model, $f_k()$ denote any smooth effects (including spatial dependence effects) and v_i are unstructured error terms (random variables).

At the time of writing, a handful of R packages allow beta regression: `betareg` [Grün *et al.*, 2011], `mgcv` [Wood, 2011] and `gamlss` [Stasinopoulos and Rigby, 2007] in the frequentist field and `Bayesianbetareg` [Marin *et al.*, 2014], `zoib` Liu and Kong [2015] and `R-INLA` (the implementation of INLA in R [Martins *et al.*, 2013]) in the Bayesian counterpart. `zoib` allows zero/one inflated beta regression but only `R-INLA` allows a wide range of flexible hierarchical models to be fitted at a user-friendly and computationally efficient environment, as we will show in the following Section.

Indeed, Bayesian hierarchical methods are becoming very popular in many fields due to the complexity of the relationships involved in natural systems [Clark, 2005]. Modelling these relationships often requires specifying sub-models inside the additive predictor that allow a suspected hidden or latent effect to be inferred that characterise these relationships.

A good example may be the use of spatial latent fields that apply distance-based functions to model the spatial dependence of the data. In these cases, the main intensity of the process is driven by a set of covariates $X\beta$, also called large-scale variation, to which a spatial term is added based on a correlation function $f_w()$ that describe the unobserved small-scale variation. Consequently, we end up with a spatial correlation model, which depends on its own hyperparameters, as part of a broader model that characterises the intensity of the process; in other words, we have a hierarchical model with a spatial latent variable.

A popular point-referenced spatial model, the geostatistical model, has the characteristic that the spatial covariance function $f_w()$ is continuous over the range of the spatial effect. Based on this function, it is customary to assume a Gaussian latent field $W \sim N(0, Q(\kappa\tau))$ with covariance matrix Q that depends on two hyperparameters, in the case of `R-INLA`, κ and τ. These hyperparameters determine the range and the variance of the spatial latent field. When we include this in the additive predictor of a beta distributed process Y, we obtain a hierarchical model with at least three stages:

- First stage: $Y|\beta, W \sim Be(\boldsymbol{X\beta} + W, \rho)$
  where Y are conditionally independent given W.
- Second stage: $W|\kappa, \tau \sim N(0, Q(\kappa\tau))$
  where W is a Gaussian latent spatial model.
- Third stage: priors on $(\beta, \rho, \kappa, \tau)$.

A common problem with this kind of hierarchical model is that there is no closed expression for the marginal posterior distributions of the parameters and hyperparameters, so numerical approximations are needed. The typical approach to approximate these posteriors is to use MCMC simulation methods. Unfortunately, MCMC can get very computationally inefficient when applied to complex models such as spatial models.

3. THE INLA APPROACH FOR GEOSTATISTICAL MODELS

Performing inference and prediction under a geostatistical Gaussian field W entail the so-called "big n problem" [Banerjee *et al.*, 2003]. This problem is related to the dense covariance matrix Q, which traduces into very high MCMC computational costs. In this vein, the stochastic partial differential equations (SPDE) approach in `R-INLA` allows reducing the required number of computations from $O(n^3)$ [Stein *et al.*, 2004] to $O(n^{3/2})$ [Cameletti *et al.*, 2013] in the two dimensional spatial domain. In what follows, we first present the INLA method followed by the SPDE approach.

The INLA algorithm, proposed by Rue *et al.* [2009], is a numerical approximation method to perform Bayesian inference. The most remarkable feature of INLA, as opposed to MCMC, is that it allows the posterior distributions of latent Gaussian models to be accurately approximated through Laplace approximations [Laplace, 1986, Tierney and Kadane, 1986], even for complex models without becoming computationally prohibitive. INLA exploits the fact that latent Gaussian models admit conditional independence properties [Rue and Held, 2005], which allows expressing them as computationally efficient Gaussian Markov random fields (GMRFs) with a sparse precision matrix [Rue and Held, 2005].

The estimation of the latent components, collected in a set of parameters $\boldsymbol{\theta} = \{\boldsymbol{\beta}, \boldsymbol{W}\}$ and hyperparameters $\boldsymbol{\Omega} = \{\rho, \kappa, \tau\}$ in `R-INLA`, is computed in three steps. First, the posterior marginal distribution of the hyperparameters is approximated by using the Laplace integration method

$$p(\boldsymbol{\Omega}|Y) \approx \left.\frac{p(Y|\boldsymbol{\theta},\boldsymbol{\Omega})p(\boldsymbol{\theta}|\boldsymbol{\Omega})p(\boldsymbol{\Omega})}{\widetilde{p}(\boldsymbol{\theta}|\boldsymbol{\Omega},Y)}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}^*(\boldsymbol{\Omega})} = \widetilde{p}(\boldsymbol{\Omega}|Y), \tag{3.1}$$

where $\widetilde{p}(\boldsymbol{\theta}|\boldsymbol{\Omega}, Y)$ is the Gaussian approximation, given by the Laplace method, of $p(\boldsymbol{\theta}|\boldsymbol{\Omega}, Y)$ and $\boldsymbol{\theta}^*(\boldsymbol{\Omega})$ is the mode for a given $\boldsymbol{\Omega}$.

Then, `R-INLA` approximates $p(\theta_i|\boldsymbol{\Omega}, Y)$ by using again the Laplace integration method

$$p(\theta_i|\boldsymbol{\Omega},Y) \approx \left.\frac{p(\boldsymbol{\theta}|\boldsymbol{\Omega},Y)}{\widetilde{p}(\boldsymbol{\theta}_{-i}|\theta_i,\boldsymbol{\Omega},Y)}\right|_{\boldsymbol{\theta}_{-i}=\boldsymbol{\theta}^*_{-i}(\theta_i,\Omega)} = \widetilde{p}(\theta_i|\boldsymbol{\Omega},Y), \tag{3.2}$$

where $\widetilde{p}(\boldsymbol{\theta}_{-i}|\theta_i, \boldsymbol{\Omega}, Y)$ is the Laplace Gaussian approximation to $p(\boldsymbol{\theta}_{-i}|\theta_i, \boldsymbol{\Omega}, Y)$ and $\boldsymbol{\theta}^*_{-i}(\theta_i, \boldsymbol{\Omega})$ is its mode. This strategy can be very computationally expensive since $\widetilde{p}(\boldsymbol{\theta}_{-i}|\theta_i, \boldsymbol{\Omega}, Y)$ has to be recomputed for each value of $\boldsymbol{\theta}$ and $\boldsymbol{\Omega}$. See section 3.2 in Rue *et al.* [2009] for a more detailed text on the different approximation approaches available in `R-INLA`.

Finally, `R-INLA` approximates the marginal posterior distributions based on the previous two steps

$$p(\theta_i|Y) \approx \int \widetilde{p}(\theta_i|\boldsymbol{\Omega},Y)\widetilde{p}(\boldsymbol{\Omega}|Y)d\boldsymbol{\Omega}, \tag{3.3}$$

where the integral can be numerically solved through a finite weighted sum applied in certain integration points and then interpolating in between. For a more detailed text on the selection of integration points see section 3.1(c) in Rue *et al.* [2009].

As mentioned above, INLA exploits the good computational properties of GMRFs to perform fast Bayesian inference. Nevertheless, continuous GFs (like the ones involved in geostatistical models) are continuously indexed, thus, in principle, not applicable in INLA. In this regard, Lindgren *et al.* [2011] provided a clever approximation of a GF with Matérn covariance function (3.4) to a GMRF using a fractional stochastic partial differential equation.

Lindgren *et al.* [2011]'s approximation of a GF requires that its covariance function is of the Matérn family. Following Lindgren *et al.* [2011]'s notation, the Matérn covariance function for an stationary and isotropic GF is

$$\mathcal{C}(d) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa||s_i - s_j||)^\nu K_\nu(\kappa||s_i - s_j||), \tag{3.4}$$

where κ is a scaling parameter that determines the effective range r of the spatial effect.

The approximation by Lindgren *et al.* [2011] fall on the fact that a GF $z(\boldsymbol{s})$ with Matérn covariance function is a solution to the linear fractional SPDE

$$(\kappa^2 - \Delta)^{\alpha/2} z(\tau \boldsymbol{s}) = \mathcal{W}(\boldsymbol{s}), \quad \boldsymbol{s} \in \mathbb{R}^d, \alpha = \nu + d/2, \kappa > 0, \nu > 0, \tag{3.5}$$

where Δ is the Laplacian, d is the dimension of the GF $z(\boldsymbol{s})$, ν is the smoothness parameter of the Matérn function and $\mathcal{W}$ is the Gaussian spatial white noise process.

Finally, the solution to the SPDE can be approximated using the Finite Element Method [Zienkiewicz *et al.*, 1977] through a deterministic basis function representation defined on a triangulation of the domain $\mathcal{D}$ (see Figure 1 for the triangulation used in the case study of the following Section). The triangulation, so-called *mesh*, of the study area is based on Delaunay triangulations [Delaunay, 1934], which, as opposed to a regular grid, allows a flexible partition of the region into triangles that can satisfy different types of constraints to better accommodate different characteristics of the study area.

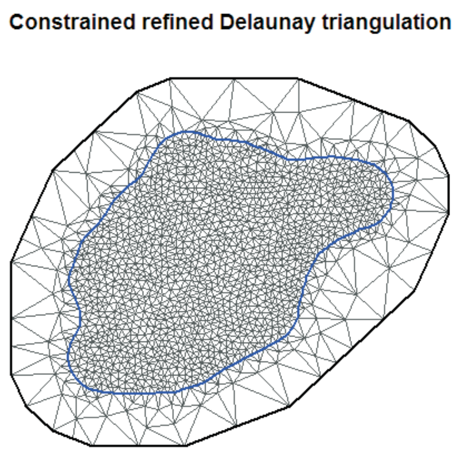


Figure 1: Triangulation of the study area. The outer ring of sparse triangles allows us avoid having a border effect inside the study area.

4. APPLICATION TO TRAWL DISCARD PROPORTIONS

The modelling approach proposed to tackle spatially sampled proportions was applied to a trawl fishery discard database in the Spanish Mediterranean. Fishery discards, i.e. the part of the catch that is thrown back to the sea dead, constitute an unnecessary biomass loss from the marine systems. A repeatedly proposed discard mitigation measure is the spatial management of fishery resources [Kelleher, 2005, Bellido *et al.*, 2011, Pennino *et al.*, 2014]. In this regard, spatial beta regression is specially important to the fishery discards framework since it allows the spatial assessment of discard proportions, which allows assessing the economic benefit of a fishing operation against its ecological impact due to the discard portion of the catch.

4.1. Data

Trawl discard data were collected according to European Comission [2009] regulation, which establishes a métier-based discard sampling programme. Specifically this study was based on bottom trawl data for the south-eastern part of the Spanish Mediterranean Sea (Figure 2) [see Pennino *et al.*, 2014, for a more detailed description of the métiers].

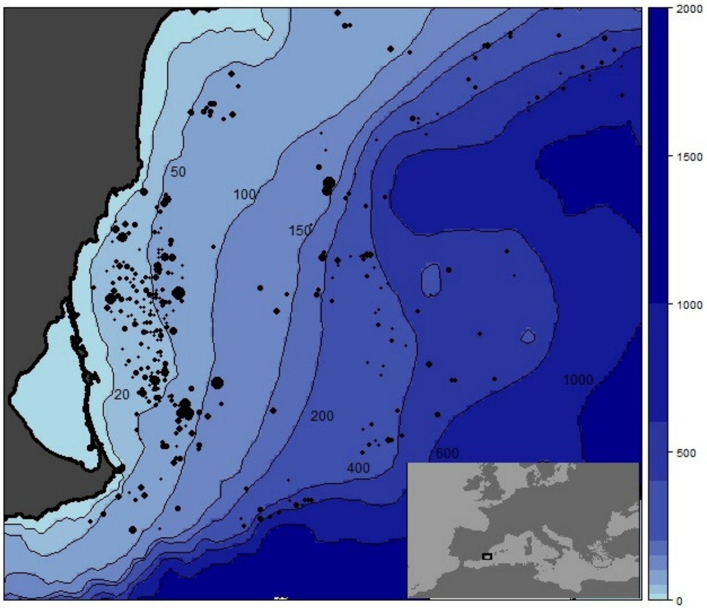


Figure 2: Map of the study area with bathymetric contours in meters. Black dots represent the centroids of the 391 sampled hauls and size plotted according to the observed discard proportion.

The database, provided by the *Instituto Español de Oceanografía* (IEO, Spanish Oceanographic Institute), contains a total of 391 hauls collected between 2009 and 2012, including catch and discard data disaggregated by species. The characteristics of each fishing operation (date, geolocation and depth) were also extracted directly from this database.

A discard proportion response variable of regulated species was created as the fraction of discarded biomass of the total catch. Unlike total discards, discard proportions represent benefit versus loss, and are therefore a better indicator to assess whether or not discards are disproportionate to the catch.

4.2. Modelling trawl discard proportions

The analysis of trawl discard proportions included the total catch of each fishing haul, the mean bathymetry of the haul, a geostatistical term and a vessel effect as predictors (Table 1). Therefore, assuming that the discard proportion Y_i at location i follows a beta distribution, the final model can be expressed as:

$$
\begin{aligned}
Y_i &\sim \mathrm{Be}(\mu_i, \phi_i), \quad i = 1, ..., n \\
\mathrm{logit}(\mu_i) &= \beta_c c_i + d_i + W_i \\
\beta_c &\sim N(0, 0.001) \\
\Delta^2 d_j &= d_j - 2d_{j+1} + d_{j+2} \sim N(0, \rho_d), \quad j = 1, ..., m \\
\log\rho_D &\sim LogGamma(0.5, 0.00005) \\
\boldsymbol{W} &\sim N(0, \mathbf{Q}(\kappa, \tau)) \\
2\log\kappa &\sim N(\mu_\kappa, \rho_\kappa) \\
\log\tau &\sim N(\mu_\tau, \rho_\tau)
\end{aligned} \tag{4.1}
$$

where the mean of discard proportions enters the model through the logit link, i indexes the location of each haul and j indexes different depths (d_j, representing the different values of bathymetry starting at $d_1 = 40$ metres till $d_{m=30} = 720$ metres). In the last two rows μ stands for the mean of the normal distributions while ρ denotes its corresponding precision.

Table 1: List of covariates included in the analysis and the effect assigned to them.

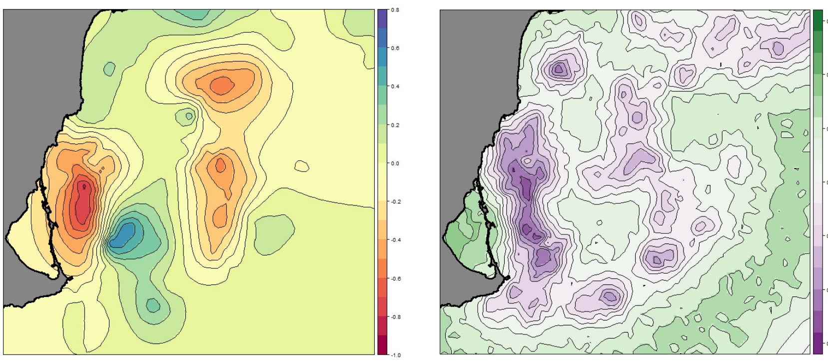
Variable	Description	Unit	Effect
Total catch	Total catch of the haul	Kilograms	Linear
Location	Geolocation	UTM	Geostatistical
Depth	Mean depth of the haul	Meters	Non-linear effect
Vessel	Sampled vessel ID	—	Random noise effect

Based on the work by Rochet and Trenkel [2005], who found that discard proportions are not fully proportional to the catch, the total catch of each haul $C = (c_1, ..., c_n)$ was introduced as a linear effect with vague normal prior distributions as implemented by default in `R-INLA`. The exploratory analysis revealed non-linear relationships between depth and the discard proportion, so a second order random walk (RW2) latent model was applied based on constant depth increments d_j. These RW2 models, which perform as Bayesian smoothing splines [Fahrmeir and Lang, 2001], can be expressed as a computationally efficient GMRF [Rue and Held, 2005], and are therefore applicable in INLA. The smoothing of the bathymetric effect was selected visually by subsequently changing its prior distribution while models were scaled to have a generalized variance equal to one [Sørbye and Rue, 2014].

The two dimensional geostatistical latent model $\boldsymbol{W}$, introduced to identify fine-scale hot-spots, depends on two hyperparameters κ and τ that define the variance and the range of the spatial effect. Specifically, and with the smoothing parameter of the Matérn (3.4) fixed ($\nu = 1$), the range of the spatial terms is approximately $\sqrt{8}/\kappa$ and the variance $1/(4\pi\kappa^2\tau^2)$. The priors for κ and τ are specified over the logτ and 2logκ. Default `R-INLA` prior distributions were used, where μ_κ is specified so that the range of the field is 20% of the longest distance in the field and μ_τ is chosen so that the mean variance of the field is one. The rest of the prior distributions in use are described in (4.1).

4.3. Results

Figure 3 shows the posterior mean and the standard deviation of the spatial component, which represents the intrinsic spatial variability of the data without the rest of the independent variables. This effect highlights (in blue), high discard proportion areas or hot-spots. Similarly, two cold-spots were found (in red), one in the coastal shallow waters in front of the lagoon and another in the mid-northern part of the 150–300 meter strata. These hot-spots characterise the areas where more discards are expected as compared to other areas with similar environmental conditions. As a consequence, a marine spatial planning framework could consider these areas for protection so that discarded/wasted biomass is minimised.



(a) Mean (b) Standard deviation

Figure 3: Posterior predictive mean and standard deviation maps of the spatial component of discard proportions.

As expected, the total catch of the haul had a positive effect on the expected discard proportions (posterior mean = 0.038; 95% CI = [0.0027, 0.0049]), i.e. the discard proportion increases with total catch increments. The bathymetric effect showed a negative relationship of discard proportions to depth, suggesting that the highest discard proportions are located in shallow waters and decrease with depth (Figure 4).

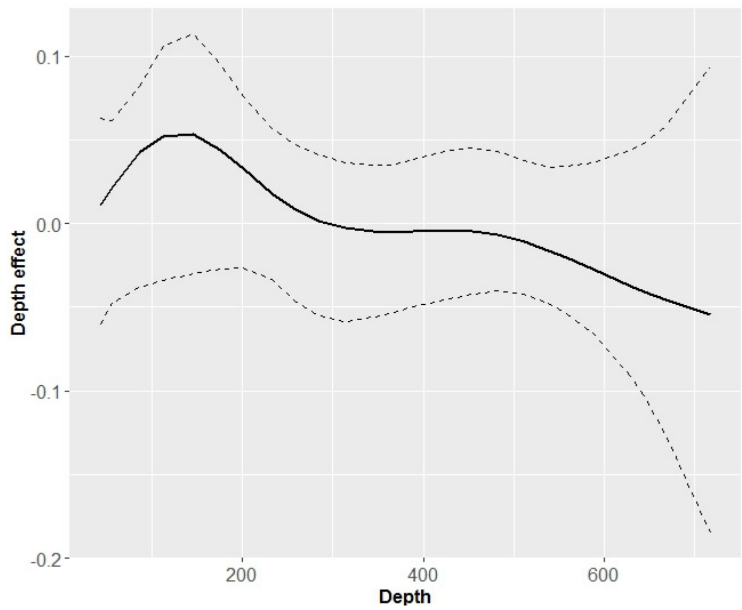


Figure 4: Marginal effect of the bathymetry in the linear predictor. The continuous line represents the mean effect and dashed lines their 95% credible intervals.

Finally, no vessel effects was identified in the study area suggesting that discard proportions are reasonably homogeneous across vessels.

5. CONCLUSIONS

In this paper, we use a Bayesian hierarchical spatial beta model to analyse spatially sampled proportion data. To this end, we use a simple reparametrisation of the beta distribution to apply regression on the mean of the process. The Bayesian approach allows a straightforward quantification of uncertainty, which is important for decision making, while the hierarchical structure allows a more natural model specification, especially when including complex latent models such as geostatistical terms.

Beta regression overcomes all the drawbacks of the traditional data transformations [Warton and Hui, 2011, Ferrari and Cribari-Neto, 2004]. First, it allows a direct interpretation of model parameters in terms of the original data; second, the analysis is not sensitive to the sample size; and lastly, posterior distributions are expected to concentrate well within the bounded range of proportions. It is only when observations on the extremes of the distribution are present, i.e. 0 and 1, that the beta distribution does not provide a satisfactory description of the data. A possible solution to this problem is to add some small value to the proportion, which introduces minimal bias while still satisfying the above criteria [Warton and Hui, 2011]; otherwise, zero and/or one inflated models may be required [Ospina and Ferrari, 2012], now available in the `zoib` package [Liu and Kong, 2015] for R.

The incorporation of spatial random effects in beta regression models can be very useful in a wide range of disciplines. For example mapping plant coverage in ecology; mapping budget allocation in econometrics; mapping the percentage of retirees in sociology, mapping sex-ratios in species, etc. Furthermore, combining the Bayesian spatial hierarchical modelling approach [Banerjee *et al.*, 2003] and the temporal extension of Da-Silva and Migon [2016], the beta regression framework can be extended to the spatio-temporal domain. Consequently, it is possible to tackle problems such as the evolution of plant epidemics [Stein *et al.*, 1994], the spatio-temporal evolution of temperature [Hengl *et al.*, 2012] or the understanding of the spatial dynamism of species over time, as in Paradinas *et al.* [2015]. It must be taken into account that the computational burden of these models can be even more demanding than in the purely spatial domain, making `R-INLA` and its SPDE module two almost necessary tools to deal with them.

The Bayesian analysis of fisheries distribution is a very important field of research in marine ecology [Muñoz *et al.*, 2013, Quiroz *et al.*, 2015]. The case study presented here applies spatial beta regression to identify fishery discard hot-spots based on discard proportions, which, as opposed to total discard units, assess the biomass benefit against the amount of wasted biomass that constitute discards. Our results have identified at least one high discard proportion hot-spot in the study area. Under a marine spatial planning framework that seeks to minimise the ecological impact of the fishing activity, the characterisation of hot-spots could be specially useful for policy makers, as it would allow them to protect those hot-spots as areas of special interest.

To conclude, we would like to mention that the geostatistical beta regression approach proposed here to analyse proportions is not only applicable to non-binomial proportional data but also to binomial proportional data, i.e. proportions measured as *x out of n*. In fact, applying beta regression in these cases may be an easier and more natural approach to avoid the usual problem of overdispersion in logistic regression than that proposed in Wilson and Hardy [2002] using GLMMs.

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