BAYESIAN ROBUSTNESS MODELLING USING THE FLOOR DISTRIBUTION *

Authors: J.A.A. Andrade
– Department of Statistics and Applied Mathematics,
Federal University of Ceara, 60455-670,
Brazil (ailton@ufc.br)

Edward Omey
– Mathematics and Statistics,
HUB, Stormstraat 2, 1000 Brussels,
Belgium, (edward.Omey@kuleuven.be)

C.T.M. Aquino
– Department of Statistics and Applied Mathematics,
Federal University of Ceara, 60455-670,
Brazil, (mendes.aquino@gmail.com)

Abstract:

- In Bayesian analysis, the prior distribution and the likelihood can conflict, in the sense that they can carry diverse information about the parameter of interest. The most common form of conflict is the presence of outliers in the data. Usually, problems of conflicts are solved by assigning heavy-tailed distributions to that source of information which may be causing the conflict. However, the class of heavy-tailed distributions is not well defined, therefore there are many ways to define heavy tails. The class O-regularly varying distributions is rather unknown in Statistics, it basically embraces those distributions whose tails decay oscillating between two power functions. In this work we study a new distribution which has this property and, as a consequence, yields robust models for location and for scale parameter models separately. We provide explicit expressions for some relevant quantities concerning the distribution, such as the moments, distribution function, etc. Besides, we show how conflicts can be resolved using this distribution.

Key-Words:

- *Bayesian robustness modelling; conflicting information; O-regularly varying distributions; heavy-tailed distributions.*

AMS Subject Classification:

- 62E15.

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1. INTRODUCTION

In Bayesian analysis, two sources of information are used to study a phenomenon of interest: the prior distribution and the likelihood function. In this way we combine, through Bayes' theorem, the evidences from both the data and some relevant subjective knowledge about the parameter of interest. However, since models are only a try to describe the reality, which is much more complex, the modelling process is inevitably subject to errors. In fact, the model misspecification can lead to wrong conclusions, since it can strongly affect the posterior distribution. The conflict of information can arise from a model not prepared to deal with diverse information, e.g. outliers, leading to different evidences about the parameters, one coming from the prior and the other coming from the likelihood. The most common form of conflict are the outliers, since they will carry information far apart from the prior distribution and the rest of the data.

This behaviour was first identified by Lindley (1968), who suggest the use of the Student-t distribution to resolve the conflicts. Dawid (1973) established conditions on the data and the prior distributions which yields robust posterior distributions for the location parameters. Several works followed this thread, basically improving Dawid's conditions as, for instance, O'Hagan (1979, 1988 and 1990) and O'Hagan & Le (1994). All these works concerned only location parameter models. In order to solve conflict of information in scale and location parameter structures separately, Andrade & O'Hagan (2006) study a class of heavy-tailed distributions different from the ones considered by Dawid (1973) and O'Hagan (1979), namely the class of *regularly varying distributions*. A more general approach for location-scale structures was proposed by Andrade & O'Hagan (2011). Andrade *et al* (2013) proposed alternative conditions for the location and the scale parameter models which are slightly easier to verify. Andrade & Omey (2013) give several new conditions using different classes of distributions, such as the subexponential and L classes. For a more complete literature review on robustness modelling, see O'Hagan & Pericchi (2012). The papers of Andrade & O'Hagan showed that, working within the regularly varying class, the outlying information will be only partially rejected, in the sense that it exerts an initial influence which, even though is constant, does not vanish as the outlier becomes large. Thus, concerning this aspect Desgagné (2013, 2015) proposes a new class of distributions which allow to resolve conflicts in scale (and location-scale) parameter(s) models by fully rejecting the conflicting information (full robustness). However, this led to rather complex conditions and distributions, which can limit the applications. Andrade & Omey (2016) proposes to use the class of *O-regularly varying distributions* (ORV), which are much more intuitive and also allows full robustness in scale parameter models.

The robustness we are treating is related to the conflict of information, the thread initiated by Dawid (1973), in which some of the sources of information (prior/likelihood) carries some information that is away from the rest of the information (See Andrade & O'Hagan, 2006).

One important thing to notice is that, although there are many new distributions which could be used for robustness modelling, for the best of our knowledge, no other distribution has this tail behaviour, with oscillating decay between two regularly varying distributions. This peculiar behaviour of the floor distribution, besides being heavy tail, also can motivate the creation of new distributions involving some sort of waving functions.

In this work we study the Floor distribution, which is in the O-regularly varying class of heavy-tailed distributions. It was firstly suggested by Andrade & Omey (2016), however here we compute all the relevant quantities of the distribution, such as moments, distribution function, random numbers generator, etc. In Section 2 we give the definitions of regular and O-regular variation. In addition we provide a rule for creating ORV distributions and show how conflicts of information can be resolved in Bayesian analysis context. In Section 3 we present the Floor distribution and its quantities. A simulation study comparing the floor distribution with the exponential one, using different proportions of outliers and sample sizes, is provided in Section 4. Section 5 provides an example in which outlying information is automatically rejected by the model as it becomes large. Finally, we conclude with some general remarks in Section 6.

2. HEAVY TAILS AND REGULAR VARIATION

Roughly speaking the concept of heavy tails is associated with those distributions whose tails decay at least slower than the function e^{-x}. However, there is not a widely spread accepted definition of heavy tail. Most of the definitions in the literature are contextualised in some area. In this work we define heavy tail as regular variation.

The concept of regular variation was introduced by Karamata (1930). Feller (1971) studied the application of such concept on probability theory and Andrade & O'Hagan (2006 e 2011) used regularly varying distributions in robust Bayesian modeling. Others studies include Landau (1911), Valiron (1913), Pólya (1917), de Haan (1970) and Seneta (1976). The main reference about regular variation used in this paper is the book written by Bingham *et al* (1987).

Definition 2.1. (Regular variation) A measurable function f is said to be regularly varying at infinity with index ρ, $\rho \in \mathbb{R}$ and $\lambda > 0$, if

$$\lim_{x\to\infty} \frac{f(\lambda x)}{f(x)} = \lambda^{\rho}. \tag{2.1}$$

We denote it by $f \in R_{\rho}$.

Particularly, if $\rho = 0$, then f is said to be a slowly varying function, denoted by $f \in R_0$. We write the set of all regularly varying functions as $R = \{R_{\rho} : \rho \in$

$(-\infty, \infty)\}$. The characterisation theorem establishes that if $f(x) \in R_\rho$, then $f(x)$ can be written as $f(x) = x^\rho \ell(x)$, where $\ell(x)$ is a slowly varying function. For more details about Karamata's theory and, in particular, regular variation, see Bingham *et al* (1987). Definition 2.1 can be interpreted as the tail behavior of a probability density function, i.e., there is a relation between regular variation and heavy tails. For example, if $f \in R_{-\rho}$ as $x \to \infty$, then the right tail decreases like a power function $x^{-\rho}$.

The O-regular variation class (which we call ORV) extends the concept of regular variation and was introduced by Avakumović (1936). This class involves distributions whose tails decrease at any behaviour between two regularly varying functions. For example, the tails of an ORV distribution can decrease oscillating between two power functions (Andrade & Omey, 2016).

Definition 2.2 (O-regular variation)**.** A probability density f is said to be O-regularly varying at *infinity*, denoted by $f \in ORV$, if f satisfies

$$\lim_{x\to\infty} \sup \frac{f(xy)}{f(x)} < \infty, \forall y > 0. \tag{2.2}$$

Along with the definition of ORV, we have the *upper* and *lower Matuszewska indexes*. If $f \in ORV$, the upper index of f is given by

$$\alpha(f) = \lim_{y\to\infty} \frac{\log \limsup_{x\to\infty} f(xy)/f(x)}{\log(y)}, \tag{2.3}$$

and the lower index of f is given by

$$\beta(f) = \alpha(1/f) = \lim_{y\to\infty} \frac{\log \liminf_{x\to\infty} f(xy)/f(x)}{\log(y)}. \tag{2.4}$$

Since the tails of ORV distributions decrease between two polynomials, it is a broad class, thus there are many ways of constructing ORV distributions. Andrade & Omey (2016) suggest a procedure to create ORV distributions. Let f be a probability density function of the form

$$f(x) = Cb(x)A(x), \tag{2.5}$$

where C is the normalizing constant, b is bounded away from zero to infinity when x tends to infinity. For the class of distributions defined in (5), it follows that:

(i) If $xA'(x)/A(x)$ is bounded, then $f \in$ ORV.

(ii) If $A(x) \in RV_{-\alpha}$, then $f \in$ ORV.

Andrade & Omey (2016) showed that the floor distribution belongs to the O-regular class. We say that the random variable X is distributed according to a floor distribution with parameter a, denoted by $X \overset{d}{\sim} \text{floor}(a)$, if its probability density function is given by

$$f(x) = C(a)\, x^{-a}\, e^{\lfloor \log x \rfloor}\, , 1 \leq x < \infty, \tag{2.6}$$

where $C(a)$ is the normalizing constant, $a > 2$ and $\lfloor . \rfloor$ is the floor function. In other words,

$$f(x) = C(a) x^{-a+1} e^{\lfloor \log x \rfloor - \log x},$$

for $x \geq 1$. Such authors also prove that the floor distribution belongs to the O-regularly varying class of distribution. Note that this satisfies (2.5), where $A(x) = x^{a-1}$ and $b(x) = e^{\lfloor \log x \rfloor - \log x}$. It is easy to see that $A(x)$ is regularly varying with index $-a+1$. Since $\lfloor \log x \rfloor \leq \log x \leq \lfloor \log x \rfloor + 1$, we also have that $-1 \leq \log x \leq 0$, which shows that $e^{-1} \leq b(x) \leq 1$. Thus we have that the floor distribution is a O-regularly varying. We also have that the Matuszewska indices for the floor distribution are given by $\alpha(f) = \beta(f) = -a+1$.

Consider $\boldsymbol{x} = (x_1, \ldots, x_n)|\theta \overset{iid}{\sim} f(x|\theta) = \theta^{-1} h(x/\theta)$, $\theta \sim p(\theta)$, and h and p bounded continuous probability densities. Following the notation in Andrade & Omey (2016), the data are partitioned in two sets, called $\boldsymbol{x}^L$ and $\boldsymbol{x}^U$, defined by $L = f(\boldsymbol{x}^L|\theta) = \prod_{i=1}^{k} h(x_i|\theta)$ and $U = f(\boldsymbol{x}^U|\theta) = \prod_{i=k+1}^{n} h(x_i|\theta)$, where $\boldsymbol{x}^L$ are the outliers. In other words, $f(\boldsymbol{x}|\theta) = \theta^{-n} \times L \times U$, and the posterior distribution is given by

$$p(\theta|\boldsymbol{x}) = \frac{\theta^{-n} \times L \times U \times p(\theta)}{\int_0^\infty \theta^{-n} \times L \times U \times p(\theta) d\theta}.$$

Andrade & Omey (2016) showed that, if the following conditions hold,

(i) $h \in ORV$ with $\alpha(h) < 0$

(ii) $\int_0^1 y^{-k\alpha - n} \times U \times p(y) dy < \infty$

(iii) $\int_1^\infty y^{-n-k\beta} \times U \times p(y) dy < \infty$

(iv) $\int_x^\infty y^{-n} p(y) dy = O(1) \Pi_{i=1}^k x_i^{\beta - \epsilon}$

then

$$0 < \liminf_{x \to \infty} \frac{p(\theta|\boldsymbol{x})}{U\ p(\theta)} \leq \limsup_{x \to \infty} \frac{p(\theta|\boldsymbol{x})}{U\ p(\theta)} < \infty \tag{2.7}$$

The result in (2.7) establishes that, as x tends to infinity, the posterior distribution will be bounded by two quantities independent of x. Thus, the posterior distribution will be based on the prior information and the observations that are not outliers.

3. THE FLOOR DISTRIBUTION

To define the location-scale floor family of distributions, let us consider the linear transformation $Z = \sigma X + \mu$ where $\mu \in \mathbb{R}$, $\sigma > 0$ and $X \overset{d}{\sim} \text{floor}(a)$.

3.1. The probability density function

Consequently, the density of Z is given by

$$h(z) = C(a)\,\frac{1}{\sigma}\left(\frac{z-\mu}{\sigma}\right)^{-a} e^{\left\lfloor \log\left(\frac{z-\mu}{\sigma}\right)\right\rfloor}\,, \mu + \sigma \leq z < \infty,$$

where $\mu \in \mathbb{R}$ and $\sigma > 0$. Figure 1 shows plots of the density for different values of a, μ, and σ.

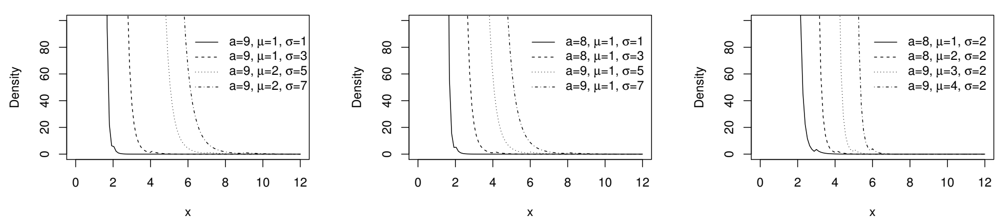

Figure 1: Floor density for different values of a, μ, e σ

3.2. The normalising constant

Note that (2.6) will be a density function if

$$\int_1^\infty C(a)\, x^{-a}\, e^{\lfloor \log x \rfloor}\, \mathrm{d}x \;=\; 1. \tag{3.1}$$

Applying the transformation $t = \log(x)$,

$$\begin{aligned}
\frac{1}{C(a)} &= \int_0^\infty e^{-a\,t}\, e^{\lfloor t\rfloor}\, e^t \,\mathrm{d}t \\
&= \int_0^\infty e^{-t(a-1)}\, e^{\lfloor t\rfloor} \,\mathrm{d}t \\
&= \int_0^1 e^{-t(a-1)}\, e^0 \,dt + \int_1^2 e^{-t(a-1)}\, e^1 \,\mathrm{d}t + \cdots \\
&= \sum_{n=0}^\infty e^n \int_n^{n+1} e^{-t(a-1)} \,\mathrm{d}t \\
&= \frac{1}{a-1}\left[\sum_{n=0}^\infty e^{-n(a-2)} - \sum_{n=0}^\infty e^{-(n+1)(a-1)+n}\right]. \qquad (3.2)
\end{aligned}$$

The two sums in (3.2) are geometric series with ratio $e^{-(a-2)}$, hence

$$C(a) = \frac{(a-1)(e^2 - e^a)}{e - e^a}, \quad a > 2.$$

3.3. Cumulative distribution function

The cumulative distribution function of $X \sim \text{Floor}(a)$ is given by:

$$F(x) \;=\; C(a) \int_1^x t^{-a}\, e^{\lfloor \log t\rfloor}\,\mathrm{d}t$$

Using the transformation $\nu = \log t$,

$$F(x) \;=\; \frac{C(a)}{a-1}\left[\sum_{n=0}^{\lfloor \log x\rfloor} e^{-n(a-2)} - e^{1-a}\sum_{n=0}^{\lfloor \log x\rfloor - 1} e^{-n(a-2)} - e^{(1-a)\log x + \lfloor \log x\rfloor}\right] \qquad (3.3)$$

The two sums in (3.3) are partial geometric series with ratios $e^{-(a-2)}$ e e^{-a}, respectively. It follows that

$$F(x) = -\frac{C(a)}{a-1}\left\{ e^{-\log(x)(a-1)+\lfloor \log(x)\rfloor} + (e^{-1}-1)\left[\frac{1-e^{-(a-2)\lfloor \log x\rfloor}}{e^{a-2}-1}\right] - 1\right\},$$

for $x > 1$ and $a > 2$. Note that the cumulative distribution involving the location and scale parameters can be obtained by simple variable transformation, that is letting $Z = \sigma X + \mu$, we have the expression $F_Z(z) = P(X \leq \frac{z-\mu}{\sigma})$.

3.4. Moment of order r

The moment of order r is calculated replacing a for $(a-r)$ in (3.1). Thus,

$$\mathbb{E}(X^r) = C(a) \int_1^\infty x^{-(a-r)}\, e^{\lfloor \log x\rfloor}\,\mathrm{d}x = \frac{C(a)}{C(a-r)}, \qquad (3.4)$$

which exists only when $r < a - 2$.

3.5. Summaries

Proposition 3.1. *If X is a random variable following the standard floor distribution with parameter a, then*

(i) *The expectation of X, given straightforwardly by (3.4), is given by* $\mathbb{E}(X) = C(a)/C(a-1),\ a \geq 3$.

(ii) *Variance. Using (3.4), the variance is*

$$Var(X) = \frac{C(a)[C^2(a-1) - C(a)C(a-2)]}{C(a-2)C^2(a-1)},\ a > 4. \tag{3.5}$$

(iii) *Quantiles can be obtained by numerically inverting the cumulative distribution function.*

(iv) *The coefficient of skewness is given by*

$$\begin{aligned}\gamma_1(X) &= \frac{C^3(a-1)C(a-2) - 3C(a)C^2(a-1)C(a-3) + 2C^2(a)C(a-2)C(a-3)}{C^{1/2}(a)C(a-3)[C^2(a-1) - C(a)C(a-2)]^{3/2}} \times \\ &\times\ C^{1/2}(a-2),\ a > 5\end{aligned}$$

(v) *It can be shown that the excess kurtosis can be written as*

$$\begin{aligned}K(X) = C(a-1)C(a-2)\{&C^3(a-1)\prod_{i=2}^{3} C(a-i) - 4C(a-4)\times \\ &\times \prod_{i=0}^{2} C(a-i) + 6C^2(a)C(a-1)\prod_{i=3}^{4} C(a-i) - 4C(a)\times \\ &\times \prod_{i=0}^{4} C(a-i) + C^2(a)\prod_{i=2}^{4} C(a-i)\}\{C(a)C(a-3)C(a-4)\}^{-1}\times \\ &\times \{C^2(a-1) - C(a)C(a-2)\}^{-2} - 3,\ a > 6\end{aligned}$$

Proof: To prove (iv), note that the Pearson's moment coefficient of skewness is given by

$$\gamma_1(X) = \frac{m_{X,3}}{\sigma_X^3},$$

where $m_{X,3} = \mathbb{E}[X - \mathbb{E}(X)]^3$, i.e., the 3th central moment of the random variable X and σ_X is the standard deviation of X. Thus, using the properties of expectation and (3.4), it can be shown that

$$m_{X,3} = \frac{C(a)[C^3(a-1)C(a-2) - 3C(a)C^2(a-1)C(a-3) + 2C^2(a)C(a-2)C(a-3)]}{C^3(a-1)C(a-2)C(a-3)}$$

Using (3.5), $[Var(X)]^{3/2} = \sigma_X^3 = \frac{C^{3/2}(a)[C^2(a-1) - C(a)C(a-2)]^{3/2}}{C^{3/2}(a-2)C^3(a-1)}$, so that the coefficient can be calculated.

To prove (v), note that the excess kurtosis is calculated by $K(X) = m_{X,4}/\sigma_X^4 - 3$. The computation of the fourth central moment is analogous to that of the coefficient of skewness, and the denominator σ_X^4 is simply the squared variance given in (3.5). □

Figure 2 shows how the expectation and variance behave as the parameter a changes. Note that as the parameter a increases, both the expectation and variance of the standard floor distribution decrease, however the variance decreases more rapidly.

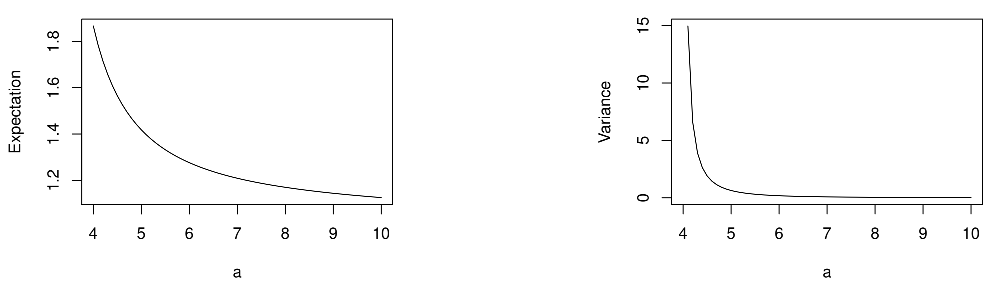

Figure 2: Expectation and variance for different values of a

Figure 3 presents the behaviour of the skewness and kurtosis coefficients as the value of a changes. Note that the value of $\gamma_1(X)$ will be always greater than $zero$, which indicates that the floor distribution is right-skewed for any value of $a > 5$.

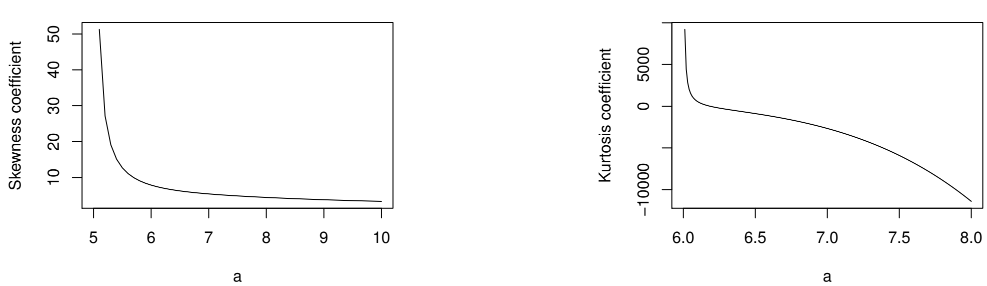

Figure 3: Skewness and kurtosis coefficients for different values of a

3.6. Summaries for the floor distribution with location and scale

Proposition 3.2. *If $Z = \sigma X + \mu$ is a random variable following the floor distribution with location and scale parameters given by μ and σ, respectively, then*

(i) *The expectation of Z, given straightforwardly by (3.4), and using properties of expectation is given by $\mathbb{E}(Z) = \sigma\mathbb{E}(X) + \mu$ $a \geq 3, \mu \in \mathbb{R}, \sigma > 0$.*

(ii) *Variance. Using (3.4), and properties of variance, we have*

(3.6)
$$Var(Z) = \sigma^2 Var(X) = \sigma^2 \frac{C(a)[C^2(a-1) - C(a)C(a-2)]}{C(a-2)C^2(a-1)}, \ a > 4.$$

(iii) *The moment of order r of the variable Z can be obtained using the binomial theorem, i.e, $\mathbb{E}(Z^r) = \sum_{k=0}^{r} \binom{r}{k} \mu^{r-k} \sigma^k \mathbb{E}(X^k)$.*

(iv) *Quantiles can be obtained by numerically inverting the cumulative distribution function.*

(v) *The coefficient of skewness is the same as of the standard floor distribution, i.e., $\gamma_1(Z) = \gamma_1(X)$.*

(vi) *The excess kurtosis coefficient is the same as of the standard floor distribution, i.e, $K(Z) = K(X)$.*

Proof: To see that the coefficient of skewness and the excess kurtosis are the same for the standard floor distribution and the floor distribution with location and scale parameter, note that $\gamma_1(Z) = \frac{m_{Z,3}}{\sigma_Z^3}$, where

$$\begin{aligned} m_{Z,3} &= \mathbb{E}\{(Z - \mathbb{E}(Z))^3\} = \mathbb{E}\{(\sigma X + \mu - (\sigma\mathbb{E}(X) + \mu))^3\} \\ &= \mathbb{E}\{\sigma^3 (X - \mathbb{E}(X))^3\} = \sigma^3 \mathbb{E}\{(X - \mathbb{E}(X))^3\} = \sigma^3 m_{X,3} \end{aligned}$$

and $\sigma_Z^3 = (\sigma_Z^2)^{\frac{3}{2}} = \{\sigma^2 Var(X)\}^{\frac{3}{2}} = (\sigma\sigma_X)^3$ Thus

$$\gamma_1(Z) = \frac{m_{Z,3}}{\sigma_Z^3} = \frac{\sigma^3 m_{X,3}}{(\sigma\sigma_X)^3} = \frac{m_{X,3}}{\sigma_X^3} = \gamma_1(X)$$

with similar computation, it can be shown that $K(Z) = K(X)$. □

3.7. Random values of the floor distribution

Through the acceptance-rejection method, values of the floor distribution are generated (Kronmal & Peterson, 1981) using the `R` software. We considered the exponential distribution with location parameter equal to 1 as the proposed distribution, so that the support of both the exponential and floor distributions are the same.

Consider the density of the floor distribution as $f(x)$ and the density of the exponential distribution as $g(y)$. The acceptance-rejection algorithm used to generate random values of a standard floor distribution with parameter a can be explicitly written as follows:

1. Generate a random value of the exponential distribution with two parameters, choosing λ so that the floor and the exponential distribution have the same expectation and fixing the location parameter as equal to 1. Call this value y.
2. Generate a random value of the uniform distribution, u.
3. If $u \leq \frac{f(y)}{cg(y)}$, set x=y (accept). Otherwise go back to step 1 to generate a new value.

In the third step, we assume that the ratio between $f(x)$ and $g(x)$ is bounded by c, a constant greater than zero.

Figure 4 shows a histogram with the distribution of 10,000 random values, generated through the acceptance-rejection method for $a = 9$, with the real density over the histogram.

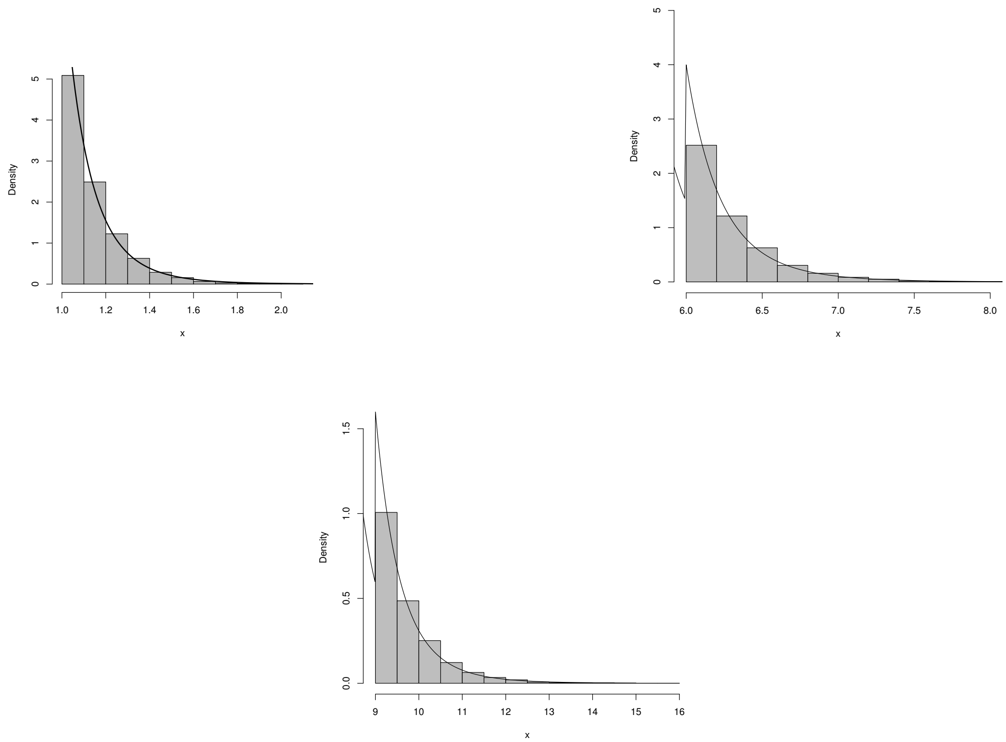

Figure 4: Histograms of generated values with $a = 9$, $\mu = 0$, $\sigma = 1$; $a = 9$, $\mu = 4$, $\sigma = 2$; $a = 9$, $\mu = 4$, $\sigma = 5$, respectively

4. SIMULATION STUDY

A simulation study was performed to compare the floor distribution to the exponential distribution, under different sample sizes and proportion of outliers. We used samples of sizes $20, 80$ and 100, and for each sample size we considered three proportions of outliers: $0.05, 0.10$ and 0.15.

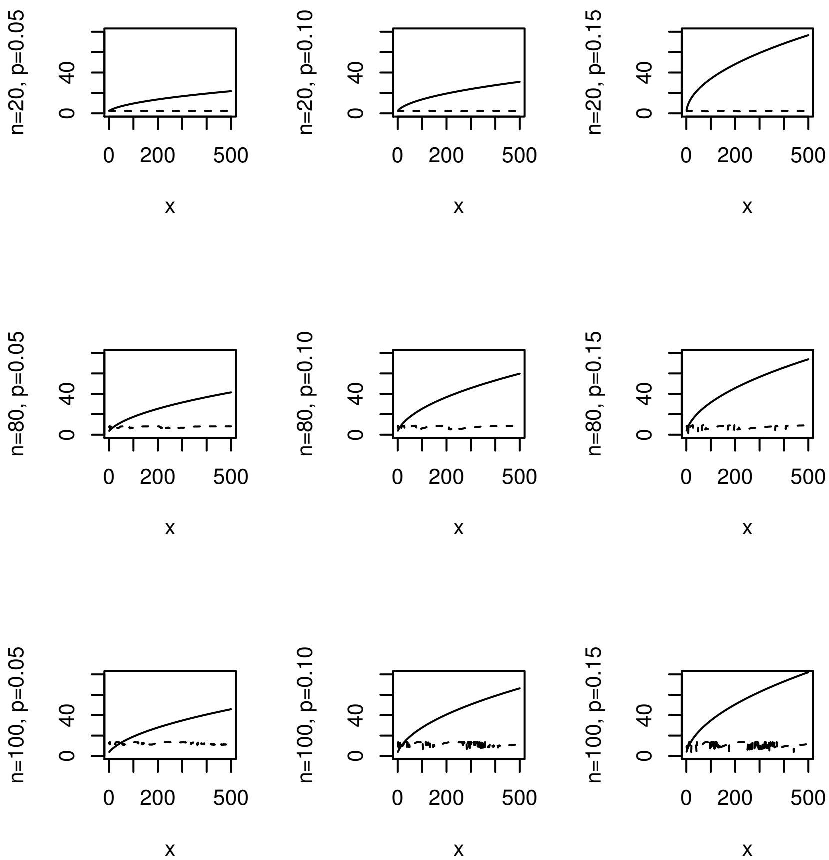

Figure 5: Comparison of location parameter posterior estimation for the floor and exponential distributions under different sample sizes and proportions of outliers.

The Figure 5 shows how the posterior estimation for the location parameter behaved as the outlying observations increased. The floor distribution is represented by the dashed line and the exponential distribution is represented by the continuous line. As the Figure 5 shows, the estimation of the location parameter under the floor distribution was not so affected as under the exponential distribution.

5. APPLICATIONS – BAYESIAN ROBUSTNESS

In this section, we illustrate how the floor distribution can resolve conflict of information by rejecting the outlying information. We compare the behaviour of the posterior estimates under a floor distribution and exponential distribution models. The usual procedure to assess robustness in a Bayesian model is to make one (or a few) observations in the data to tend to infinity, and check how the posterior estimates behaves. Thus, modelling accordingly to conditions (i)-(iv) which lead to the result (2.7), the posterior distribution will automatically reject the conflicting observation.

We use the data from Kapur & Lamberson (1977, p. 240), which refers to X: the number cycles to failure (in ten thousands) for 20 heater switches subject to an overload voltage. Following the same methodology used in Andrade & Omey (2016), the data are modelled with two different densities with scale parameter: exponential, with density given by $f(x|\theta) = \theta^{-1}e^{-x/\theta}, x > 0$ and floor, with density given by

$$f(x|\theta) = \frac{C(a)}{\theta}\left(\frac{x}{\theta}\right)^{-a} e^{\log\lfloor x/\theta\rfloor},$$

for $x > \theta$. The conditions established by Andrade & Omey (2016) consider location and scale parameter structures separately, thus a more complex structure, involving location and scale parameters, will change their conditions, hence more investigation is required in order to assess the behaviour of the posterior quantities in the location-scale parameter case, likewise to Andrade & O'Hagan (2011).

Tahir & Saleem (2011) considered a elicited prior density is which is quite informative, in the sense that the prior variance is relatively small. Thus, $\theta \overset{d}{\sim}$ Gama$(8.9936, 21.5698)$, which we will also use for both the floor and exponential models.

We used the package `OpenBugs` with zeros trick, since the floor is a new distribution. As a result, Figure 6 was created by simulating the posterior distribution for each model, as one of the observations tends to infinity. In the case of the exponential distribution, as the outlier becomes distant from the other observations, the posterior mean is affected by the outlying information. This does not happen with the floor distribution, and thus we can see from this example that the floor distribution is robust to outliers.

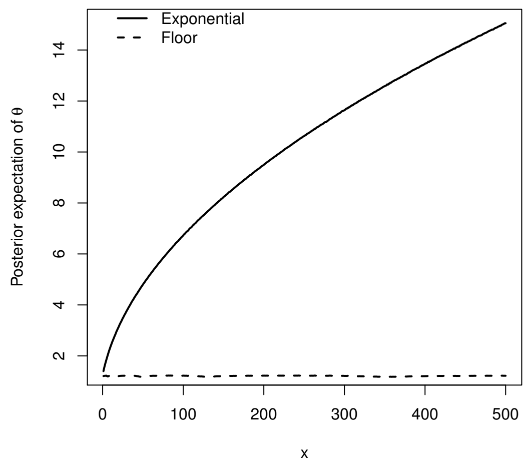

Figure 6: Posterior estimates for θ

6. DISCUSSION

In Bayesian context, robustness modelling is becoming of high interest, mainly to address problems due to misspecification of the model. In fact, in a rigorous modelling process, a researcher may change the model after detecting conflicting information such as outliers, untrusted prior information, etc. Therefore, it is important to know about the properties of the heavy-tailed distributions in order to model conveniently to resolve such conflicts. However, the large range of heavy-tailed distributions leads to a great variety of behaviours of the posterior distribution in the presence of conflict, some heavy-tailed distribution will yield robust models only for the location parameter, whereas other classes of such distributions will resolve conflicts in both location and scale parameters. In addition, different classes can lead to different ways to resolve the conflict. For instance, as pointed out by Andrade & O'Hagan (2006), in the scale parameter case, the regularly varying distributions will allow only a partial rejection of the conflicting information, whereas the class proposed by Desgagné (2013, 2015) achieve full rejection. In this work, we follow the proposal of Andrade & Omey (2016), in which uses the ORV class, which also lead to complete rejection the outlier, however the ORV class is much more intuitive and easy to work than that proposed by Desgagné (2013).

On the other hand, there are very few ORV distributions in the literature, further work should propose new ORV distributions. The floor distribution is an alternative to the exponential distribution, and is an example of how the tails can oscillate leading to a new sort of heavy-tailed distributions, which has direct applications in Bayesian Robustness.

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