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ESTIMATION OF THE FALSE ALARM PROBABILITY BASED ON THE SUCCESSIVE OBSERVATIONS OF THE SYSTEM

ESTIMAÇÃO DA PROBABILIDADE DE FALSO ALARME A PARTIR DE OBSERVAÇÕES SUCESSIVAS DO SISTEMA

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ABSTRACT:

- In this paper, we propose a recursive algorithm to obtain estimations of the false alarm probability in discrete linear systems with uncertain observations. We use a Bayesian approach, by specifying a Beta prior distribution for the unknown parameter and considering a quadratic loss function. By means of successive approximations of mixture distributions, we obtain a recursive algorithm which provides approximations for the Bayes estimators of the false alarm probability.

KEY-WORDS:

- *Uncertain Observations. False Alarm Probability.*

***RESUMO*:**

- Neste artigo, propomos um algoritmo que estima de forma recursiva a probabilidade de falso alarme em sistemas lineares discretos com observações incertas. Utilizamos uma abordagem Bayesiana, em que se especifica uma distribuição *a priori* Beta para o parâmetro desconhecido e se considera uma função perda quadrática. A estimação é feita através de aproximações sucessivas de distribuições mistura, obtendo-se aproximações para os estimadores de Bayes para a probabilidade de falso alarme.

***PALAVRAS-CHAVE*:**

- *Observações Incertas, Probabilidade de falso alarme.*

1. INTRODUCTION

In many practical situations, such as communication systems, there may be a nonzero probability (*false alarm probability*) that any observation consists of noise alone; this may be caused by an intermittent failure in the observation mechanisms. These situations can be described by an observation equation including not only an additive noise, but also a multiplicative noise component, modelled by a sequence of Bernoulli random variables; such systems are called *Systems with Uncertain Observations.*

The linear state estimation problem in linear systems with uncertain observations, under the hypotheses of mutual independence of the noises and the initial state and independence of the Bernoulli random variables, was treated by Nahi (1969). Later on, García-Ligero et al. (1997) and Caballero et al. (2000) obtained the quadratic and polynomial filters, respectively. In all these works it is assumed that, at any time, the false alarm probability or, equivalently, the probability that the signal exists in the observations, is known.

In this paper, we consider a linear discrete-time system with uncertain observations in which the uncertainties are governed by a sequence of independent Bernoulli random variables, and we assume that the probability that the observations contain the state process is unknown, but fixed throughout the time. We assume that the initial state and the additive noises of the system are gaussian and, also, that the initial state and all the noises are mutually independent.

Our aim is to obtain a recursive algorithm to estimate the false alarm probability, based on the successive observations of the system. For our purpose, we use a Bayesian approach; due to an ever-increasing computational complexity as a result of the uncertainty in the observations, the Bayes estimators of the probability are unfeasible in practice. For this reason, it becomes necessary to find approximations which are viable from a computational viewpoint.

Firstly, by using successive approximations of gaussian mixtures, we propose a method to compute approximations for the posterior densities of the unknown probability, given the observations. These approximations have also a mixture form and, hence, their computation involves an additional complexity which depends on the selected prior density. Then, we consider a Beta as the prior density and, by means of the new approximations of mixture distributions, we propose a recursive algorithm which allows us to obtain estimations of the unknown probability.

The proposed estimators of the unknown false alarm probability can be used for adapting the linear, quadratic and polynomial filtering algorithms established in Nahi (1969), Garcia-Ligero et al. (1997) and Caballero et al. (2000), respectively.

2. PROBLEM STATEMENT

Consider a discrete-time linear system with uncertain observations

$$x_{k+1} = A_k x_k + w_k, \quad k \geq 0$$

$$z_k = u_k C_k x_k + v_k, \quad k \geq 0$$

where x_k is the $n \times 1$ state vector, z_k is the $m \times 1$ observation vector at time k, and A_k , C_k are known matrices of appropriate dimensions.

The initial state, x_0, and the additive and multiplicative noises, $\{w_k;\ k \geq 0\}$ $\{v_k;\ k \geq 0\}$ and $\{u_k;\ k \geq 0\}$, satisfy:

- x_0 is a gaussian vector with zero mean and covariance matrix Σ_0.
- $\{w_k; k \geq 0\}$ is a gaussian white sequence with zero means and covariance matrices Q_k.
- $\{v_k; k \geq 0\}$ is a gaussian white sequence with zero means and covariance matrices R_k.
- $\{u_k; k \geq 0\}$ is a sequence of independent Bernoulli random variables with $P[u_k = 1] = p,$ for all $k \geq 0,$ being p an unknown parameter.
- The initial state x_0 and the noises $\{w_k;\ k \geq 0\}$, $\{v_k;\ k \geq 0\}$ and $\{u_k;\ k \geq 0\}$ are mutually independent.

Our aim is to obtain estimators for the probability p, based on the successive observations, $z_0, \ldots, z_k$, of the system, that can be obtained recursively. For this purpose, we use a Bayesian approach and, so, the problem is to obtain $\bar{p}_k = E\{p / Z^k\}$, the Bayes estimator of the probability p given the observations $Z^k = \{z_0, \ldots, z_k\}$, for a specific prior density and assuming a quadratic loss function.

To solve this problem we need to obtain the posterior density given the observations, for the selected prior density. Denoting by $f(p / Z^{-1})$ the prior density for $p,$ the posterior density, $f(p / Z^k),$ can be obtained from the Bayes theorem, and it becomes

$$f\left(p/Z^k\right)=\frac{f\left(z_k/p,Z^{k-1}\right)f\left(p/Z^{k-1}\right)}{\int f\left(z_k/p,Z^{k-1}\right)f\left(p/Z^{k-1}\right)dp},\quad k\geq 0$$

where

$$f\left(z_k/p,Z^{k-1}\right)=pf_1\left(z_k/Z^{k-1}\right)+(1-p)f_0\left(z_k/Z^{k-1}\right)$$

with $f_i\left(z_k/Z^{k-1}\right)=f\left(z_k/u_k=i,Z^{k-1}\right)$ and $f_i\left(z_0/Z^{-1}\right)=f\left(z_0/u_0=i\right),\ i=0,1.$

Accordingly, the computation of the posterior densities $f\left(p/Z^k\right)$ requires that the densities $f_i\left(z_k/Z^{k-1}\right)$, for $i=0,1$, should be calculated. From the independence hypotheses on the system, the density $f_0\left(z_k/Z^{k-1}\right)$ agrees with that of the observation noise vector v_k, that is, it is the density of the gaussian distribution $N(0,R_k)$. However, as a result of the uncertainty in the observations, the determination of $f_1\left(z_k/Z^{k-1}\right)$ is not simple and its computation grows in complexity as k increases. To avoid this difficulty, it seems natural to consider approximations for these densities. So, approximations for the posterior densities and, consequently, for the Bayes estimators of the parameter p are obtained.

We propose to approach this problem by approximating mixtures of gaussian distributions by gaussian distributions with their corresponding parameters. This task will be carried out in the next section.

3. ESTIMATION OF THE FALSE ALARM PROBABILITY

As we have commented in the above section, our first aim will be to obtain approximations, $\tilde{f}_1\left(z_k/Z^{k-1}\right)$, which avoid the ever-increasing computational complexity of the density $f_1\left(z_k/Z^{k-1}\right)$. These approximations will provide, in their turn, approximations for the posterior density, $\tilde{f}\left(p/Z^k\right)$, and for the Bayes estimators of p, $\tilde{p}_k=\int p\tilde{f}\left(p/Z^k\right)dp.$

At the first step, since $f_1\left(z_0/Z^{-1}\right)$ corresponds to the gaussian distribution $N\left(0,C_0\Sigma_0C_0^T+R_0\right)$, the density $f\left(p/Z^0\right)$ can be easily computed, providing the estimator $\overline{p}_0$.

Next, we observe that $f_1\left(z_1 / Z^0\right)$ is determined by $f\left(x_0, w_0, z_0 / Z^{-1}\right)$ and

$$f\left(x_0, w_0, z_0 / Z^{-1}\right) = \overline{p}_{-1} f_1\left(x_0, w_0, z_0 / Z^{-1}\right) + \left(1 - \overline{p}_{-1}\right) f_0\left(x_0, w_0, z_0 / Z^{-1}\right)$$

where $f_i\left(x_0, w_0, z_0 / Z^{-1}\right) = f\left(x_0, w_0, z_0 / u_0 = i\right)$, for $i = 0, 1$, is the density of the gaussian distribution

$$N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_0 & 0 & i\Sigma_0 C_0^T \\ 0 & Q_0 & 0 \\ iC_0\Sigma_0 & 0 & iC_0\Sigma_0 C_0^T + R_0 \end{pmatrix}\right).$$

Then, we approximate $f\left(x_0, w_0, z_0 / Z^{-1}\right)$ by the density of the gaussian distribution

$$N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_0 & 0 & \overline{p}_{-1}\Sigma_0 C_0^T \\ 0 & Q_0 & 0 \\ \overline{p}_{-1}C_0\Sigma_0 & 0 & \overline{p}_{-1}C_0\Sigma_0 C_0^T + R_0 \end{pmatrix}\right).$$

From this distribution, using the system equations, we obtain that $\tilde{f}_1\left(z_1 / Z^0\right)$ is the density of the distribution $N\left(C_1\hat{x}_{1/0},\ C_1\Sigma_{1/0}C_1^T + R_1\right)$, where

$$\begin{aligned} \hat{x}_{1/0} &= A_0 K_0 z_0 \\ K_0 &= \overline{p}_{-1}\Sigma_0 C_0^T \Pi_0^- \\ \Pi_0 &= \overline{p}_{-1} C_0 \Sigma_0 C_0^T + R_0 \end{aligned}$$

and

$$\Sigma_{1/0} = A_0 \Sigma_{0/0} A_0^T + Q_0$$

$$\Sigma_{0/0} = \Sigma_0 - K_0 \Pi_0 K_0^T .$$

Finally, the replacement of $\tilde{f}_1\left(z_1 / Z^0\right)$ in the expression of $f\left(p / Z^1\right)$, together with $f\left(p / Z^0\right)$, provides an approximation, $\tilde{f}\left(p / Z^1\right)$, and, from it, we obtain $\tilde{p}_1$.

The proposed procedure provides a recursive method for obtaining the densities $\tilde{f}_1\left(z_{k+1} / Z^k\right)$. In fact, let us assume that, for an arbitrary $k \geq 1$, the following approximation holds

$$f\left(x_k, w_k, z_k / Z^{k-1}\right) = \tilde{p}_{k-1} f_1\left(x_k, w_k, z_k / Z^{k-1}\right) + \left(1 - \tilde{p}_{k-1}\right) f_0\left(x_k, w_k, z_k / Z^{k-1}\right)$$

where $f_i\left(x_k, w_k, z_k / Z^{k-1}\right) = f\left(x_k, w_k, z_k / u_k = i, Z^{k-1}\right)$, for $i = 0,1,$ is the density of the gaussian distribution

$$N\left(\begin{pmatrix} \hat{x}_{k/k-1} \\ 0 \\ iC_k \hat{x}_{k/k-1} \end{pmatrix}, \begin{pmatrix} \Sigma_{k/k-1} & 0 & i\Sigma_{k/k-1}C_k^T \\ 0 & Q_k & 0 \\ iC_k\Sigma_{k/k-1} & 0 & iC_k\Sigma_{k/k-1}C_k^T + R_k \end{pmatrix}\right).$$

As in the first step, we approximate this mixture by the density of the gaussian distribution

$$N\left(\begin{pmatrix} \hat{x}_{k/k-1} \\ 0 \\ \tilde{p}_{k-1} C_k \hat{x}_{k/k-1} \end{pmatrix}, \begin{pmatrix} \Sigma_{k/k-1} & 0 & \tilde{p}_{k-1}\Sigma_{k/k-1}C_k^T \\ 0 & Q_k & 0 \\ \tilde{p}_{k-1}C_k\Sigma_{k/k-1} & 0 & \Pi_k \end{pmatrix}\right)$$

where $\Pi_k = \tilde{p}_{k-1} C_k \Sigma_{k/k-1} C_k^T + R_k + \tilde{p}_{k-1}\left(1 - \tilde{p}_{k-1}\right) C_k \hat{x}_{k/k-1} \hat{x}_{k/k-1}^T C_k^T .$

As a consequence, $\tilde{f}_1\left(z_{k+1} / Z^k\right)$ will be the density of the gaussian distribution $N\left(C_{k+1}\hat{x}_{k+1/k},\ C_{k+1}\Sigma_{k+1/k}C_{k+1}^T + R_{k+1}\right)$, where

$$\begin{aligned} \hat{x}_{k+1/k} &= A_k \hat{x}_{k/k-1} + A_k K_k \left[z_k - \tilde{p}_{k-1} C_k \hat{x}_{k/k-1}\right] \\ K_k &= \tilde{p}_{k-1} \Sigma_{k/k-1} C_k^T \Pi_k^- \end{aligned}$$

and

$$\begin{aligned} \Sigma_{k+1/k} &= A_k \Sigma_{k/k} A_k^T + Q_k \\ \Sigma_{k/k} &= \Sigma_{k/k-1} - K_k \Pi_k K_k^T . \end{aligned}$$

The approximation $\tilde{f}_1\left(z_{k+1} / Z^k\right)$ provides $\tilde{f}\left(p / Z^{k+1}\right)$ and, from this, we obtain the estimators $\tilde{p}_{k+1}$.

This procedure provides a method for the computation of $\tilde{f}\left(p/Z^{k}\right)$ and the estimators $\tilde{p}_{k}$. However, its application presents an additional difficulty, due to the fact that this posterior density also has a mixture form. Obviously, the difficulty at the computation depends on the selected prior distribution. In the following section, we consider a *Beta* as the prior distribution, and we propose a new approximation for the Bayes estimators of p.

4. ESTIMATORS APPROXIMATION

Since p is the parameter of the Bernoulli random variables $\{u_k;\ k \geq 0\}$, and the Beta family is conjugated for the sampling of that distribution, let us specify $f\left(p/Z^{-1}\right) \equiv \beta(\alpha_0, \beta_0)$. Then $\bar{p}_{-1} = \alpha_0(\alpha_0+\beta_0)^{-1}$, and

$$f\left(p/Z^{0}\right) = \delta_0 \beta(\alpha_0+1, \beta_0) + (1-\delta_0)\beta(\alpha_0, \beta_0+1)$$

where $\delta_0 = \dfrac{\bar{p}_{-1} f_1\left(z_0/Z^{-1}\right)}{\bar{p}_{-1} f_1\left(z_0/Z^{-1}\right) + (1-\bar{p}_{-1}) f_0\left(z_0/Z^{-1}\right)}$.

So, as a result of the mixture form of $f\left(p/Z^{0}\right)$, and since each posterior density is mixture of two distributions, $\tilde{f}\left(p/Z^{k}\right)$ will be a mixture of 2^{k+1} distributions.

In order to avoid the computational complexity, we propose to approximate $f\left(p/Z^{0}\right)$ by $\hat{f}\left(p/Z^{0}\right)$, the density of the distribution $\beta(\alpha_0+\delta_0, \beta_0+1-\delta_0)$, and for the subsequent steps we proceed in an analogous way. So, if

$$\hat{f}\left(p/Z^{k-1}\right) = \beta\left(\alpha_0 + \sum_{i=0}^{k-1}\hat{\delta}_i, \beta_0 + \sum_{i=0}^{k-1}\left(1-\hat{\delta}_i\right)\right)$$

where $\hat{\delta}_i = \dfrac{\hat{p}_{i-1}\hat{f}_1\left(z_i/Z^{i-1}\right)}{\hat{p}_{i-1}\hat{f}_1\left(z_i/Z^{i-1}\right) + (1-\hat{p}_{i-1}) f_0\left(z_i/Z^{i-1}\right)}$, the posterior distribution of p given Z^{k} will be mixture of two Beta distributions, with mixture parameter $\hat{\delta}_k$, and will be approximated by the distribution

$$\beta\left(\alpha_0 + \sum_{i=0}^{k}\hat{\delta}_i, \beta_0 + \sum_{i=0}^{k}\left(1-\hat{\delta}_i\right)\right).$$

The main advantage of these approximations is that the estimators can be obtained by the following recursive relation

$$\hat{p}_k = \hat{p}_{k-1} - \frac{1}{\alpha_0 + \beta_0 + k + 1}\left[\hat{p}_{k-1} - \hat{\delta}_k\right] \; k \geq 0$$

$$\hat{p}_{-1} = \frac{\alpha_0}{\alpha_0 + \beta_0}.$$

5. NUMERICAL EXAMPLE

To test the effectiveness of the proposed estimators, we have considered the following scalar system

$$x_{k+1} = 0.5x_k + w_k, \; k \geq 0$$
$$z_k = u_k x_k + v_k, \; k \geq 0$$

where the initial state, x_0, is a random variable with distribution $N(0,1)$, and the noises $\{w_k;\; k \geq 0\}$ and $\{v_k;\; k \geq 0\}$are gaussian white sequences with zero means and variances $E\{w_k^2\} = E\{v_k^2\} = 19/3$. The multiplicative noise $\{u_k;\; k \geq 0\}$ is a sequence of independent Bernoulli variables with unknown parameter p.

We have obtained numerical simulations for the observations of this system, considering different values of the parameter p ($p = 0.25$, $p = 0.5$ and $p = 0.75$) and, in each case, we have performed one hundred iterations of the proposed algorithm by assuming as prior distribution a Beta, $\beta\left(\sqrt{1+19/3}, \sqrt{19/3}\right)$; the parameters of this prior distribution specify the standard deviation of the first observation when the false alarm probability is zero and one, respectively.

The successive estimations of p, obtained by using the observations simulated with each value of the false alarm probability, are displayed in the below table and figures. A slow but clear decreasing and increasing tendency of the estimations can be noticed in the extreme cases, $p = 0.25$ and $p = 0.75$, respectively. In the case $p = 0.5$, since the prior estimation is very close to this value, we observe that the estimations are stabilised about it.

$p = 0.25$		$p = 0.5$		$p = 0.75$	
0.51831723324638	0.42251511734729	0.51831723324638	0.50839443315680	0.51831723324638	0.61270200596641
0.51692290437721	0.42123250358735	0.51624961308656	0.50881219639288	0.51780346913122	0.61255257876996
0.51008203131525	0.42067813657507	0.50790711705153	0.50746135975527	0.57376233220683	0.61854195262212
0.49792392924330	0.42059293230680	0.49635647191756	0.50576474230569	0.55830090888667	0.62076022231370
0.48977821354975	0.42003494831626	0.48609452342568	0.50421881359340	0.55280550095145	0.61872533935391
0.48294188029693	0.41851909475381	0.47725348546051	0.50328468346594	0.56874950435060	0.61721285895589
0.47418586368559	0.42137809765677	0.47040435212003	0.50309212789447	0.56020695149390	0.61621926555911
0.47084120586004	0.42241612164985	0.47476570826775	0.50196950260317	0.55204606851969	0.61668642159040
0.46703800606882	0.42163454979915	0.46727768368744	0.50453183456854	0.54701650238569	0.61767842465579
0.46099993258587	0.42441856302212	0.46089249551017	0.50646294790495	0.54049551891840	0.61597573738515
0.46107784505358	0.42319650016718	0.46641594336362	0.50543740162250	0.54663850631360	0.61480463031140
0.45472842375341	0.42171651899885	0.46605779494621	0.50449306147154	0.56270817235801	0.61488946941091
0.46955780885823	0.42032478224086	0.48127877160113	0.50327494332868	0.55853522670033	0.61456731903296
0.46317466717575	0.41897627158681	0.47522552414406	0.50186756253584	0.56564225256456	0.61470061295716
0.45796823523771	0.41786976437692	0.47011565780925	0.50160779961060	0.57599774731777	0.61348414584386
0.46199153354518	0.41863358999901	0.46886835871281	0.50071859294673	0.58469305260024	0.61213360387475
0.45736902312049	0.41765313244278	0.46405540469460	0.50112565888034	0.58438413276868	0.61138454485833
0.45352496399865	0.41643917322505	0.45960355419658	0.50208466880606	0.58851501593358	0.61017427956407
0.45226902640002	0.41576622143065	0.45557175786672	0.50056858441507	0.58584963283698	0.61257072600714
0.44815155826277	0.42115710574440	0.45199695325787	0.50238433665269	0.59307360811065	0.61095565602835
0.44654690067308	0.42020790947848	0.46194253289223	0.50090055646227	0.59762970227892	0.60986274975476
0.44582606943595	0.41917971201091	0.46082917707688	0.49969650500197	0.60460406963169	0.60983527075147
0.44250351452456	0.41845648688386	0.48035411576262	0.49887014604613	0.60603871931980	0.60887998250087
0.43905692380759	0.41724143885485	0.47589204492577	0.49984751318801	0.60825022581519	0.60944778589369
0.43942617880233	0.41625511874150	0.47941416453953	0.49973114043850	0.60500353419828	0.61027094601079
0.43797274378768	0.42031315704781	0.47588312075814	0.49846933766466	0.61421042422349	0.61050294881053
0.43483985819290	0.41916582236500	0.48041776533840	0.49772900506546	0.62385703099807	0.61110230710716
0.43183861756302	0.41854231044352	0.47720970078000	0.49752264538021	0.62444651964067	0.61347090918771
0.43078886751338	0.41895171813775	0.48551738946555	0.49820647399445	0.62398040402280	0.61196680772215
0.42960746395327	0.42132040401052	0.48233856475159	0.50334908657268	0.62438833657372	0.61105790539918
0.42676231716834	0.42030220538901	0.47945579743599	0.50251814524632	0.62260359429456	0.60990614249121
0.42415405937514	0.41987540671512	0.49102973294518	0.50127955271572	0.62387691282616	0.61016437140367
0.42164440242958	0.41927982374542	0.50361296866707	0.50020913912989	0.62129214720905	0.61008743230091
0.42096808050841	0.41914710004728	0.51164214977775	0.49915619699071	0.61880303725354	0.61443360282785
0.42118136551663	0.41817617569893	0.50841302267218	0.49916769902649	0.61637775459192	0.61667971271557
0.42178006540035	0.41838206124766	0.51327852682774	0.49844655227867	0.61474547676763	0.61942384653282
0.42183229484749	0.41737067649262	0.51048666936469	0.50067435077089	0.61305369487088	0.61882321966852
0.41983587045855	0.41643181688604	0.50952907178538	0.49982395153334	0.61155840092122	0.61791817230560
0.41829142946926	0.41605100798565	0.50885414141747	0.49933598718989	0.61752688293941	0.61705231344732
0.41694097593400	0.41691107656105	0.50646092083373	0.49825175922758	0.61813380060184	0.61612449467778
0.41906590950883	0.41947601859812	0.50431026074839	0.49756009255887	0.61634372027683	0.61780790385322
0.41793395897366	0.41828570969286	0.50628176563788	0.49778010837073	0.61466947929366	0.61669583416555
0.41593875592986	0.41771687302669	0.50472521056793	0.49719737098230	0.61281501490532	0.61643038684791
0.41486127448008	0.41769621688851	0.50970265338092	0.49705731813328	0.61092169420173	0.61754234657963
0.41800730267086	0.42030717424191	0.50773531588242	0.49744963824493	0.61597628271750	0.62054625633693
0.41980836763697	0.41993716379386	0.50825266187492	0.50067156353097	0.61392214854909	0.61934401777018
0.42728667798995	0.41891917157318	0.50670755978178	0.49943897568785	0.61284796987762	0.61846077869030
0.42538764641818	0.41846015479370	0.50701107913350	0.50076677831059	0.61187769334657	0.61763840968242
0.42347656534294	0.41829165612604	0.50708962191031	0.49973329741485	0.61211890302355	0.61878927763336
0.42444278421478	0.41734900510805	0.51218307264665	0.49894997686286	0.61206970263562	0.62039930264733
0.42333939490793	0.41726390448502	0.50991528635783	0.49800126976457	0.61065995693767	0.62250188891052

Table: Estimations of *p* with prior distribution $\beta\left(\sqrt{1+19/3}, \sqrt{19/3}\right)$

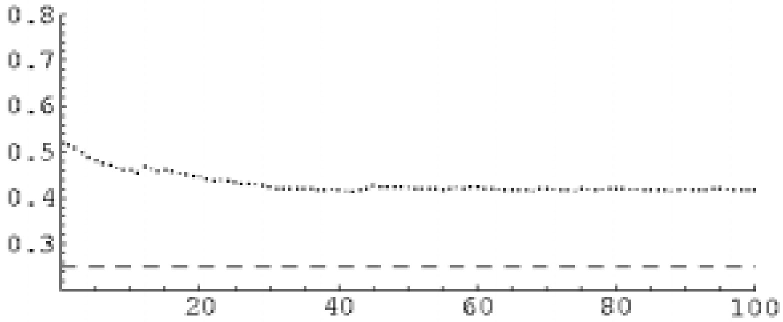

Figure 1. Estimations of $p = 0.25$

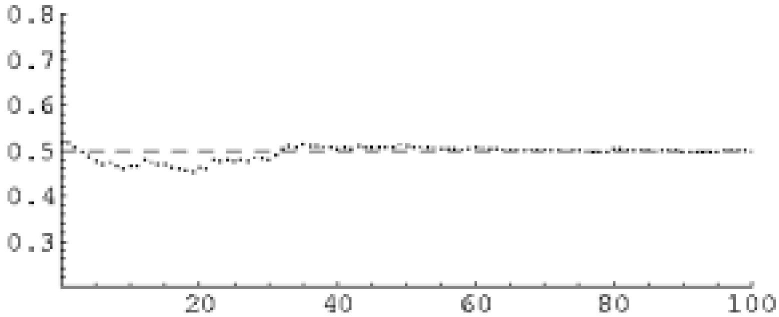

Figure 2. Estimations of $p = 0.5$

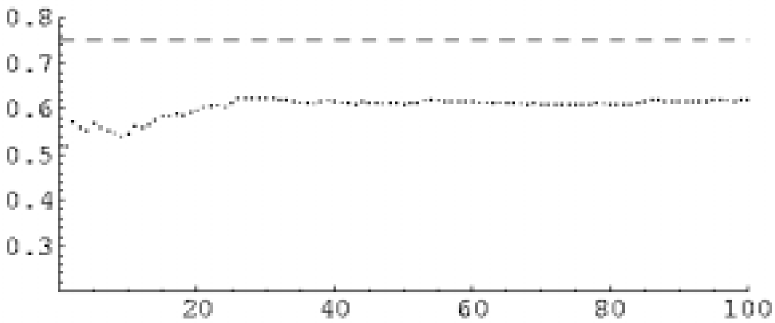

Figure 3. Estimations of $p = 0.75$

6. CONCLUSIONS

In this paper, we consider a system with uncertain observations in which the initial state and noises are mutually independent and the probability that the observations contain the state process is unknown, but fixed throughout the time. We propose a recursive estimation algorithm for that probability, based on the successive observations of the system.

The estimators obtained with this algorithm are approximations to the Bayes estimators of the false alarm probability when a Beta prior distribution is specified and a quadratic loss function is considered. These approximations are obtained by means of successive approximations of mixture distributions.

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